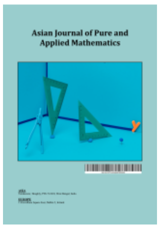

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Fixed Points for "O-W-C"- Self-Mappings on C-M-Spaces

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Author's contribution

The sole author designed, analysed, interpreted and prepared the manuscript.

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Original Research Article

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Abstract

In the present research article, we obtained a unique common fixed points for "O-W-C"(Occasionally Weakly Compatible) self-mappings with satisfying a generalized contractive type condition on "C- M-Space"(Cone –Metric-Space). Our main aim to extends, improves and generalizes recent comparable existing theorems in the references.

Keywords: C-M-Space(Cone Metric Space); fixed point; common fixed point; fixed point theorem; O-W-C(Occasionally Weakly Compatible).

AMS -2010 Subject Classification: "54H25" and "47H10".

1 Introduction

In the Non-linear analysis, the study of fixed point results are very most important tool. And the concept of a cone metric space was introduced by the authors "Huang" and "Zhang" [1]. And they have been generalized a metric space concept, that is, cone metric space, for this they have been replaced the "real numbers" by an

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ordered "Banach" space and also got some of the common fixed point results on "C-M-Space"(Cone – Metric-Space). Later on, many authors has been generalized, improved and extended "Huang" and "Zhang" [1] results in so many forms(for e. x.,[2-6],[7-13]). And recent times "Bhatt" and "Chandra" [6] have got some fixed point results in "O-W-C" (Occasionally Weakly Compatible) self-mappings on C-M-Spaces (Cone –Metric- Spaces). Our main aim is in this research article we proved fixed point result for "O-W-C"(Occasionally Weakly Compatible) self-mappings on "C- M-Spaces".

2 Preliminaries

In ordered to get fixed point results, we need some of the basic Lemmas & Definitions which are in [1,7].

2.1 Definition

Suppose that $S_1$is a real "Banach" space. And a subset T_1 of S_1 is said to be a cone iff

(A) $T_1$is not empty and is closed and $T_1 \neq \{0\}$;

(B) $\alpha_1, \beta_1 \in R$, $\alpha_1 ,\beta_1 \geq 0$, $u_1, v_1 \in T_1$implies $\alpha_1 u_1 + \beta_1 v_1 \in T_1$;

(Ci) $T_1 \cap (-T_1) = \{0\}$.

For a "cone"$T_1 \subset S_1$, defining a partial order "$\leq$" with respect to $T_1$by

"$\alpha_1 \leq \beta_1 \Leftrightarrow \beta_1 - \alpha_1 \in T_1$". A cone T_1 is said to be "normal" if there is a number $M_1 > 0$such that for all $\alpha_1, \beta_1 \in T_1$,

"$0 \leq \alpha_1 \leq \beta_1 \Rightarrow ||\alpha_1|| \leq M_1 ||\beta_1||$".

The smallest +vee number satisfying the above inequality is called the "normal constant" of T_1, while" $\alpha_1 << \beta_1$" stands for "$\beta_1 - \alpha_1$" interior of T_1.

2.2 Definition

Supposethat $X_1$is a non-empty set of S_1. And suppose that the map

$\rho: X_1 \times X_1 \rightarrow S_1$ satisfying the following:

(A). $0 \leq \rho(\alpha_1, \beta_1)$ for all $\alpha_1,\beta_1 \in X_1$ and $\rho(\alpha_1, \beta_1) = 0$ if and only if $\alpha_1 = \beta_1$;

(B). $\rho(\alpha_1,\beta_1) = \rho(\beta_1, \alpha_1)$ for all $\alpha_1, \beta_1 \in X_1$;

(C). $\rho(\alpha_1,\beta_1) \leq \rho(\alpha_1, \gamma_1) + \rho(\gamma_1, \beta_1)$ for all $\alpha_1, \beta_1, \gamma_1 \in X_1$.

Then $\rho$is called a "C-Metric "(Cone -Metric) on $X_1$and (X_1,ρ) is said to be a "C-M-Space" (Cone –Metric-Space).

2.3 Definition

Let (X_1,ρ) be a C-M-Space. We said that $\{x_n\}$ is a

(A). Cauchy sequence if for every $b_1 \epsilon_1$ with "$b_1 >> 0$", then , $\exists$ a natural number $n_1 \ni$

$\rho(x_n,x_m) << b_1$, for all n, m> n_1.

(B). convergent sequence if for every "$b_1 \epsilon S_1$" with"$b_1 >> 0$", then ∃ a natural number N_1∋

$\rho(x_n, x_1) << b_1$, for all n> $n_1$for some fixed x_1 in X_1.And denote this "$x_n \rightarrow x_1$", as $n \rightarrow \infty$.

2.4 Definition

A C-M- Space (X_1,ρ) is complete if every "Cauchy"sequenceisa convergent in X_1.

2.5 Definition

Suppose that two self- mappingsP_1 and $Q_1$are in aset X_1. And if "$w_1 = P_1x_1 = Q_1x_1$" for some $x_1 \epsilon X_1$, then x_1 is called a "coincidence point" of P_1 and Q_1, and then w_1 is said to be a "point of coincidence" of P_1 and Q_1.

2.6 Proposition

Suppose thattwo self-mappings P_1 and $Q_1$are "O-W-C"(Occasionally –Weakly-Compatible)in a set X_1iff there is a point "x_1 in X_1" which is a"coincidence point" of P_1 and Q_1 at which P_1 and Q_1 are "commute".

2.7 Lemma

Suppose that two self-mappingsP_1, Q_1 are (in X) "O-W-C"(Occasionally Weakly Compatible) of X_1. If P_1 and Q_1 have a unique point of" coincidence" "$w_1 = P_1x_1 = Q_1x_1$", then "w_1" is a unique common "fixed point "of P_1 and Q_1.

3 Main Results

In this main part, we prove a "unique commonfixed point"result for "O-W-C"(Occasionally Weakly Compatible) self - mappings in "C-Metric-Space "(Cone Metric Space).

Our "main theorem" is follows:

3.1 Theorem

Suppose thatthat p_1 and q_1 are two self-mappings of X_1 in a "C-M- Space"

(X_1,ρ) and $S_1$is a "normal cone" . And satisfying the following:

(i) $\rho(p_1x_1, q_1y_1) \leq \lambda_1 \, Max\{\rho(q_1x_1,q_1y_1), \rho(p_1x_1, q_1x_1)+\rho(p_1y_1, q_1y_1) /2\}+$
$\lambda_2 \, \text{Max}\{ \rho(q_1x_1, q_1y_1),\rho(q_1x_1, p_1x_1) +\rho(q_1y_1, p_1y_1)/2 \}$,
for all $x_1,y_1 \epsilon X_1$, where λ_1, $\lambda_2 > 0$ and$\lambda_1 + \lambda_2 < 1$.

(ii) $p_1(X_1) \subset q_1(X_1)$

(iii) $p_1$and q_1 are O-W-C.

Then p_1 and $q_1$are having"unique common fixed point" in X_1.

Proof: By (iii) p_1 and q_1 are "O-W-C". Then there exists a point $\alpha_1 \epsilon X_1$ such that

$p_1\alpha_1 = q_1\alpha_1 = w_1$and there exists another point $\beta_1 \epsilon X_1$ for which $p_1\beta_1 = q_1\beta_1 = u_1$.

Now we claim that : $p_1\alpha_1 = q_1\beta_1$. Suppose that $w_1 \neq u_1$. Then from (i) we get that

$$
\begin{aligned}
\rho(p_1\alpha_1, q_1\beta_1) \leq\ & \lambda_1\, Max\{\rho(q_1\alpha_1, q_1\beta_1)\ , \rho(p_1\alpha_1, q_1\alpha_1) + \rho(p_1\beta_1, q_1\beta_1)\ /2\}+ \\
& \lambda_2\, \mathrm{Max}\{ \rho({}_1q_1\alpha_1, q_1\beta_1), \rho(q_1\alpha_1, p_1\alpha_1) + \rho(q_1\beta_1, p_1\beta_1)/2 \}\ , \\
\\
\leq\ & \lambda_1\, Max\{\rho(p_1\alpha_1, p_1\beta_1), \rho(p_1\alpha_1, p_1\alpha_1) + \rho(p_1\beta_1, p_1\beta_1)\ /2\}+ \\
& \lambda_2 \mathrm{Max}\{\rho(p_1\alpha_1, p_1\beta_1), \rho(p_1\alpha_1, p_1\alpha_1) + \rho(p_1\beta_1, p_1\beta_1)/2\ \}, \\
\\
\leq\ & \lambda_1\, Max\{\rho(p_1\alpha_1, p_1\beta_1)\ ,0\}+ \lambda_2\, \mathrm{Max}\{ \rho(p_1\alpha_1, p_1\beta_1),0\ \} \\
\leq\ & (\ \lambda_1 + \lambda_2)\rho(p_1\alpha_1, p_1\beta_1) \\
<\ & \rho(p_1\alpha_1, p_1\beta_1),
\end{aligned}
$$

since $\lambda_1 + \lambda_2 < 1$. Which is a contradiction.

Therefore, $\rho(p_1\alpha_1, p_1\beta_1) = 0$.

Implies that, $p_1\alpha_1 = p_1\beta_1$. Therefore, $p_1\alpha_1 = q_1\alpha_1 = p_1\beta_1 = q_1\beta_1 = w_1 = u_1$.

That is, $p_1\alpha_1 = q_1\alpha_1 = w_1$. Hence, "$w_1$"is a unique point of coincidence. By the (2.1)Lemma , we get that "w_1" is the "unique common fixed point " of p_1 and q_1. And this completes the proof of the theorem.

4 Conclusion

In this present paper, our main results are more improve and general results than the existing results in [6].

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Competing Interests

Author has declared that no competing interests exist.

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