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Advancing Numerical Methods of Block Multi-Derivative Approaches for ODEs of Various Orders

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Authors' contributions

This work was carried out in collaboration among all authors. All authors read and approved the final manuscript.

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Abstract

This work presents numerical methods of block multi-derivative approaches for ordinary differential equations (ODEs) of Various Orders. The derivation of the methods is achieved by applying the techniques of interpolation and collocation to a power series polynomial, which is considered an approximate solution to the problems. Higher derivative terms are introduced to improve the accuracy of the method, giving room to modify the method for solving second and third-order initial value problems (IVPs) of ordinary differential equations (ODEs). Details conformation of the block method is presented, showing that the method is zero stable, consistent and convergent. The method is applied block-by-block to first, second and third-order initial value problems (IVPs) of ordinary differential equations. The application of the method to a real-life example also yields accurate results.

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1 Introduction

In many fields, like engineering and management, mathematical models are created to understand real-world behavior, often resulting in differential equations. These equations are used to study various phenomena such as the movement of planets, the decay of radioactive elements, and changes in species populations. These models often involve non-linear equations that usually do not have analytical solutions, so numerical approximation methods are necessary.

This paper considered the initial value problems (IVPs) of ordinary differential equations (ODEs) of the form

$$\left.\begin{array}{l} y'(x) = f(x, y(x)) \qquad y(x_0) = y_0 \\ y''(x) = f(x, y(x), y'(x)) \qquad y(x_0) = y_0 \quad y'(x_0) = y_0' \\ y'''(x) = f(x, y(x), y'(x), y''(x)) \qquad y(x_0) = y_0 \quad y'(x_0) = y_0' \quad y''(x_0) = y_0'' \end{array}\right\} \tag{1}$$

The solution of (1) has been extensively discussed in the literature using different approaches. Lambert [1], Brugnano and Trigiante [2], Fatunla [3], among others. The authors considered the reduction approach of higher-order ODEs, where the problems are reduced to an equivalent system of first-order ODEs, after which appropriate methods are applied. This reduction increases the problem's dimension, leading to more computational difficulties. Awoyemi and Idowu [4], Akinfenwa and Jator [5], Olabode and Momoh [6],[7], Awoyemi [8], Ramos and Momoh [9] successfully applied numerical algorithms as integrators to solve higher order-order initial value problems directly without reducing it to first order (ODEs).

Two major classes of numerical methods are usually adopted for the numerical solution of equation (1). The classes are Runge-Kutta and Linear multistep method see Ramos and Momoh [9]. More work has been done on applying RK methods for solving first-order ODEs, and their direct application to higher-order ODEs still needs to be improved. Recently, authors have concentrated their effort on numerical solutions of second and third-order ODEs using linear multistep methods. This effort has led to the introduction of a block mode for resolving implicit linear multistep methods, which is a clear departure from the well-known predictor-corrector approach. According to Olabode [10], the block method was introduced by Milne [11] as a starting step for the predictor-corrector pairs has been identified to be costly since their subroutine for incorporating the stating values leads to lengthy computational time, Jator [12], Kayode *et el.*, [13], Olabode and Momoh [6].

The following are some of recent work on block method for directly solving initial value problems of ODEs: Olabode and Momoh [6],[7], Adeyefa [14], Ramos and Momoh [9], Duromola *et el.*, [15], Olabode *et el.*, [16], Kashkari [17], Ogunfeyitimi and Ikhile [18]. This work aims to make available a single method that can handle the first-order initial value problems of ODEs and the direct solution of second and third-order initial value problems of ODEs. Works in this category include Adeyefa and Olagunju [19] and Adeyefa [14]. This paper is organized as follows: section one gives an introduction, section two discusses the derivation of the new method, and section three analyzes the method's basic properties. Section four considers numerical examples, while section five discusses results. Finally, section six gives conclusions.

2 Derivation of the Method

Power series polynomial of the form

$$y(t) = \sum_{j=0}^{k} a_j t^j \tag{2}$$

is allowed to approximate $y(t)$ in (1) to derive a numerical method for solving the equation. The coefficients $a_j's$ are to be determined, t is continuous and differentiable within the interval of integration $[a, b]$ and k represents the summation of the numbers of interpolation and collocation points. The first, second and third derivatives of (2) give the following:

$$y'(t) = \sum_{j=1}^{3} j a_j t^{j-1} \tag{3}$$

$$y''(t) = \sum_{j=2}^{3} j(j-1) a_j t^{j-2} \tag{4}$$

and

$$y'''(t) = \sum_{j=3}^{3} j(j-1)(j-2) a_j t^{j-3} \tag{5}$$

Interpolating (2) at t_n and collocating (3)-(5) at $t = t_{n+1}$ where $t_{n+i} = t_n + ih$ and setting $t_n = 0$, yields

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 2h & 3h^2 \\ 0 & 0 & 2 & 6h \\ 0 & 0 & 0 & 6 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} y_n \\ f_{n+1} \\ g_{n+1} \\ q_{n+1} \end{bmatrix} \tag{6}$$

The determinant of the matrix is 12 12 hence, matrix X is non-singular. The parameters $aj's$ are obtained by using Crammer's Rule as follows;

$$a_0 = y_n \tag{7}$$

$$a_1 = \frac{1}{2}\left(2f_{n+1} - 2hg_{n+1} + h^2 q_{n+1}\right) \tag{8}$$

$$a_2 = \frac{1}{2}\left(g_{n+1} - hq_{n+1}\right) \tag{9}$$

$$a_3 = \frac{1}{6} q_{n+1} \tag{10}$$

Substituting $(7) - (10)$ into (2) with $t = t_{n+t}$ gives a continuous formula of the form

$$y(t) = \alpha_0(t) y_n + h(\beta_1(t) f_{n+1}) + h^2(\gamma_1(t) g_{n+1}) + h^3(\mu_1(t) q_{n+1}) \tag{11}$$

with the following coefficients,

$$\alpha_0(t) = 1, \tag{12}$$

$$\beta_1(t) = (t), \tag{13}$$

$$\gamma_1(t) = \left(\frac{t^2}{2} - t\right), \tag{14}$$

$$\mu_1(t) = \left(\frac{t}{2} - \frac{t^2}{2} + \frac{t^3}{6}\right). \tag{15}$$

Evaluating (11) at $t = 1$, gives the discrete scheme

$$y_{n+1} = y_n + hf_{n+1} - \frac{1}{2}h^2 g_{n+1} + \frac{1}{6}h^3 q_{n+1} \tag{16}$$

Equation (16) are the formulas that constitute the proposed method for solving first order ODEs. One of the novelty of this work is the adoption of the derived method for directly solving second and third-order ODEs. To adopt Equation (16) for the solution of second and third-order ODEs, two additional formulas are obtained by evaluating the first and second derivative of Equation (11) at t_n, which gives

$$f_n = f_{n+1} - hg_{n+1} + \frac{1}{2}h^2 q_{n+1} \tag{17}$$

$$g_n = g_{n+1} - hq_{n+1} \tag{18}$$

Equation (16) and (17) are combined for solving second order ODEs which equation (16), (17) and (18) are combined for direct solution of third-order ODEs

3 Basic Properties of the Method

This section shall the analysis of the basic properties of the method which include order, zero stability, consistency, convergency and region of absolute stability.

3.1 Order and error constant

Equation (16) - (18) are discrete schemes belonging to the class of LMM of the form

$$\sum_{j=0}^{k} \alpha_j y_{n+j} = h \sum_{j=0}^{k} \beta_j f_{n+j} + h^2 \sum_{j=0}^{k} \gamma_j g_{n+j} \tag{19}$$

According to Fatunla [3] and Lambert [1], local truncation error associated with equation (16) - (18) is define by the operator

$$L[y(t); h] = \sum_{j=0}^{k} \alpha_j y(t_n + jh) - h^2 \beta_j y''(t_n + j) - h^3 \gamma_j y'''(t_n + j) \tag{20}$$

where $y(t)$ is an arbitrary function, continuously differentiable on [a,b]. Expanding (20) in Taylor series about the point t, and collecting the like terms in h yields

$$L[y(t); h] = c_0 y(t) + c_1 h y^1(t) + c_2 h^2 y^2(t) + c_3 h^3 y^3(t) + \cdots + c_p h^p y^p(t) \tag{21}$$

where the $c_0, c_1, c_2, ..., c_{p+1}$ are obtain as

$$c_0 = \sum_{j=0}^{k} \alpha_j, c_1 = \sum_{j=1}^{k} j\alpha_j, c_2 = \frac{1}{2!} \sum_{j=1}^{k} j^2 \alpha_j, c_q = \frac{1}{q!} \left[\sum_{j=1}^{k} j^2 \alpha_j - q(q-1) \sum_{j=1}^{k} \beta_j j^{q-2} \right] \tag{22}$$

according to Lambert [1], equation (16) -(18) is of order p if $c_0 = c_1 = c_2 = k, c_p = 0$ and $c_{p+r} \neq 0$ for $r = 1$ for the case of first-order ODEs and $r = 2, 3$ for second and third-order respectively. The $c_{p+r} \neq 0$ is called the error constant and $c_{p+r} h^{p+r} y^{p+r}(t_n)$ is the principal local truncation error at the point t_n. Thus, the block (16)-(18) is of order $p = 3$ and error constants $c_{p+r} = [-\frac{1}{24}, \frac{1}{6}, \frac{1}{2}]^T$

3.2 Zero-stability of the method

The general form of block method is given as

$$A^0 Y_m = A^r Y_{m-1} + h^\rho [B^i F_m + B^0 F_{m-1}] \tag{23}$$

A method is said to be zero stable, if the roots

$$det[\lambda A^0 - A^r] = 0 \tag{24}$$

of the first characteristic polynomial satisfied $|\lambda| \leq 1$ and the roots with $|\lambda| = 1$, the multiplicity must not exceed the order of the differential equation Fatunla [3]. This kind of stability concerned the behaviour of the numerical method as $h \to 0$, the derived method given the system of equations that can be written as;

$$A^0 Y_m = A^r Y_{m-1} \tag{25}$$

Case 1: For first order ODEs

where A^0 is identity matrix. Consider (16) and setting $h = 0$, we have

$$\begin{bmatrix} 1 \end{bmatrix} \begin{bmatrix} y_{n+1} \end{bmatrix} - \begin{bmatrix} 1 \end{bmatrix} \begin{bmatrix} y_n \end{bmatrix} = \begin{bmatrix} 0 \end{bmatrix}$$

$\rho(\lambda) = \lambda \begin{bmatrix} 1 \end{bmatrix} - \begin{bmatrix} 1 \end{bmatrix}$
$\lambda - 1 = 0$

Since $\lambda_1 = 1$, the block methods are zero stable.

Case 2: For second order ODEs
where A^0 is identity matrix. Consider (17) and setting $h = 0$, we have

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} y_{n+1} \\ y'_{n+1} \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} y_n \\ y'_n \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\rho(\lambda) = \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}$$

$\lambda(\lambda - 1) = 0$

Since $\lambda_1 = 1$, the block methods are zero stable.

Case 3: For third order ODEs
where A^0 is identity matrix. Consider (18) and setting $h = 0$, we have

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} y_{n+1} \\ y'_{n+1} \\ y''_{n+1} \end{bmatrix} - \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} y_n \\ y'_n \\ y''_n \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\rho(\lambda) = \lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

$\lambda^2(\lambda - 1) = 0$
Since $\lambda_1 = 1$, the block methods are zero stable.

3.3 Consistency

A method is considered consistent if it has an order greater than one Henrici [20]. From the above analysis, it shows that the derived method is consistent. Since the order $p = 3 > 1$.

3.4 Convergency

The necessary and sufficient condition for an LMM to be convergent is that it must be consistent and zero stable Henrici [20]; hence, the methods are convergent since the derived method has order $p = 3$, zero-stable and consistent.

3.5 Region of absolute stability

The area in the complex z-plane in which the numerical method exhibits the behaviours of the real solution is known as the region of absolute stability (see Ramos and Momoh [9]). The method's behaviour is studied here by considering its application to test equations.

$$y' = -\lambda y \tag{26}$$

This yields

$$BY_n = GY_{n-1} \tag{27}$$

where $Y_n = (y_{n+1})^T$, $\quad Y_{n-1} = (y_n)^T$ and $\quad z = h\lambda$

$$B = \left(\frac{1}{6}(z+3)z^2 + z + 1)\right) \tag{28}$$

$$G = \begin{pmatrix} 0 & -1 \end{pmatrix} \tag{29}$$

The amplification matrix

$$M(h) = B^{-1}G \tag{30}$$

$$M(h) = \begin{pmatrix} \frac{1}{6}(z+3)z^2 + z + 1 \end{pmatrix}^{-1} \begin{pmatrix} 0 & -1 \end{pmatrix} \tag{31}$$

where the dominant eigenvalue $\left(\frac{1}{6}(z+3)z^2 + z + 1\right)$ is a function of z

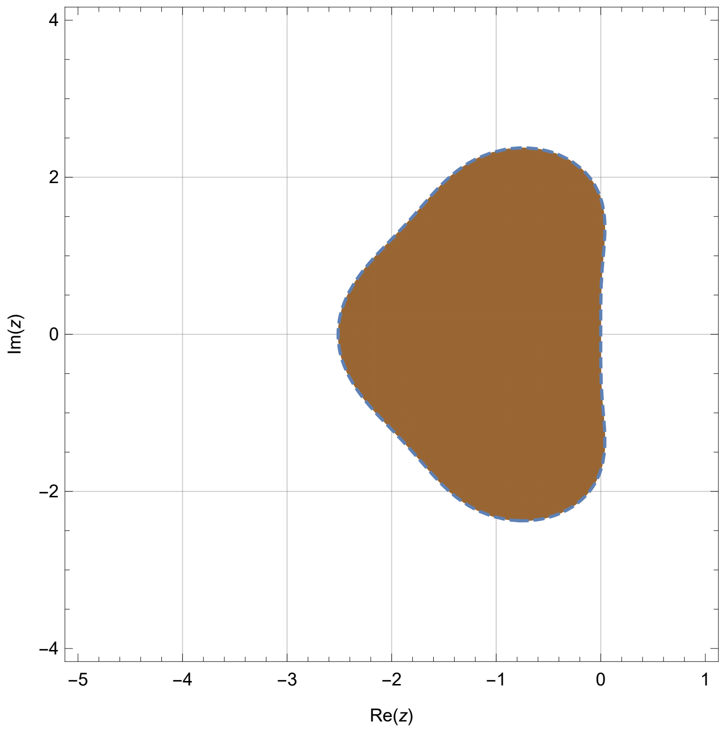

Fig. 1. Region of absolute stability of the Method

From the Fig. 1 above, it is observed that the method exhibit a Runge-Kutta like region of absolute stability

4 Numerical Examples

The performance of the methods is tested on four numerical examples

Problem 1:

Consider an Artificial Intelligent (AI) system integration into a hardware setup, like a server or a robotic unit, where maintaining an optimal temperature is crucial for performance and longevity. The system has a cooling mechanism (like fans or liquid cooling), and the effectiveness of this cooling changes as the system's temperature approaches the operating temperature. Let $T(t)$ denotes the temperature of the AI system at time t. The difference between the current temperature and the desired optimal temperature could influence the temperature change rate. A simple first-order differential equation could be:

$$\frac{dT}{dt} = -k.(T(t) - T_{opt})$$

where:
. $\frac{dT}{dt}$ is the rate of change of the system's temperature over time.
. k is a positive constant representing the cooling system's efficiency.
. $T(t)$ is the present temperature at time t.
. T_{opt} is the optimal operating temperature for the AI system.

Interpretation

. when the system's temperature $T(t)$ is much higher than T_{opt}, the cooling system works effectively, leading to a faster decrease in temperature ($\frac{dT}{dt}$).
. As $T(t)$ approaches T_{opt}, the cooling system's effectiveness reduces (as less cooling is needed), leading to a slower rate of temperature change.
.The model captures the balancing act of the cooling system: it actively cools the system when the temperature is far from optimal and reduces its cooling effect as the temperature nears the desired level, preventing over cooling.
The model

$$\frac{dT}{dt} = -k(T - T_{opt})$$

has exact solution

$$T(t) = T_o e^{-k(t)} - (e^{-kt} - 1)T_{opt}$$

where

$$T_o = T(t_o)$$

Given that
$T_{opt} = 24^o c$; $\quad T_o = T(0) = 18^o c$ and $k = 0.8$ (i.e 80% cooling efficiency)

$$\frac{dT}{dt} = -0.8(T(t)24)$$

where

$$T_o = T(0) = 18$$

has exact solution

$$T(t) = 18e^{-0.8(t)} - (e^{-0.8t} - 1)24$$

Table 1. Solution of Problem 1 for k=1, p=3 and h=0.01

x-value	y - exact	y-computed	Error
0.1	18.461301921680	18.461301912288	9.392415734055 E-09
0.2	18.887137266203	18.887137248862	1.734058940883 E-08
0.3	19.280232833601	19.280232809590	2.401106513616 E-08
0.4	19.643105777558	19.643105748004	2.955334821309 E-08
0.5	19.978079723786	19.978079689685	3.410147542127 E-08
0.6	20.287299649163	20.287299611388	3.777554624662 E-08
0.7	20.572745616907	20.572745576224	4.068309777949 E-08
0.8	20.836245455742	20.836245412821	4.292027000474 E-08
0.9	21.079486464240	21.079486419667	4.457294977556 E-08
1.0	21.304026215297	21.304026169579	4.571779754770 E-08

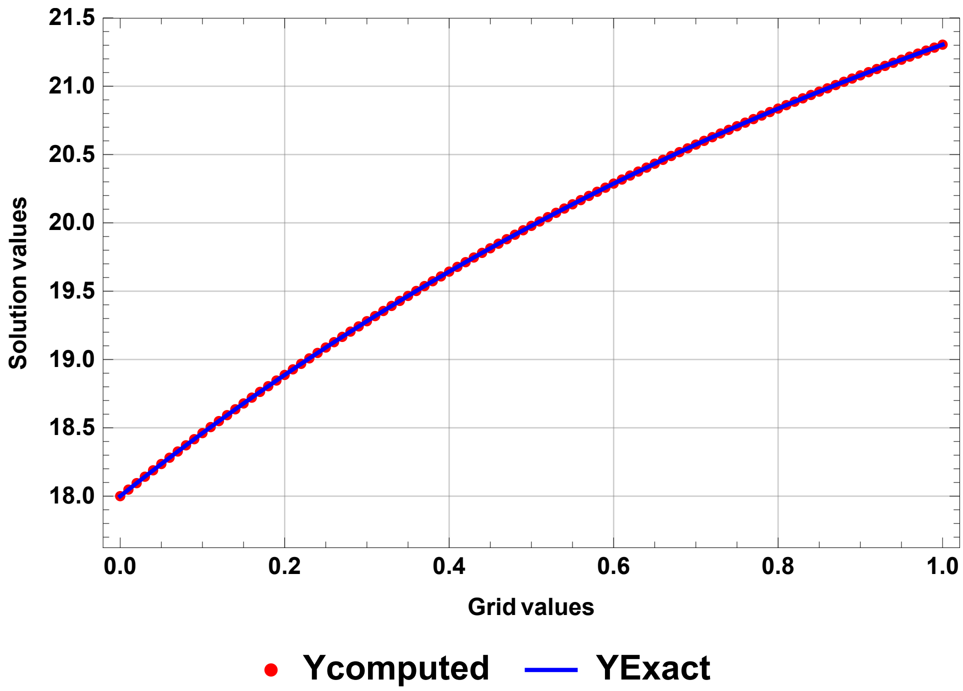

Fig. 2. Graphical Solution of Problem 1 for k= 1, p= 3 and h= 0.01

Problem 2

Consider the linear initial value problem of ODEs
$y'' = y'$
$y(x) = 1 - e^x, \quad y(0) = 0, \quad y'(0) = -1, \quad h = 0.01$

Table 2. Comparison of Error for Problem $2(k = 1,\ p = 3$ **and** $h = 0.01)$

x-value	y - exact	y=computed	Error	Ogunware & Ezekiel [21] $k = 1, p = 8, h = 0.01$
0.1	-0.010050167084	-0.010050167083	9.478690351 E-13	5.831942458 E-12
0.2	-0.020201340027	-0.020201340023	3.785581178 E-12	1.087659387 E-11
0.3	-0.030454533954	-0.030454533945	8.551027165 E-12	1.511754063 E-11
0.4	-0.040810774192	-0.040810774184	1.528266847 E-11	1.853811766 E-11
0.5	-0.051271096376	-0.051271096352	2.401954492 E-11	2.112140771 E-11
0.6	-0.061836546545	-0.061836546511	3.480128250 E-11	2.285023525 E-11
0.7	-0.072508181254	-0.072508181207	4.766810111 E-11	2.370716438 E-11
0.8	-0.083287067675	-0.083287067612	6.266082257 E-11	2.367449502 E-11
0.9	-0.094174283705	-0.094174283625	7.982087859 E-11	2.273425941 E-11
1.0	-0.105170918076	-0.105170917976	9.919031899 E-11	2.086821844 E-11

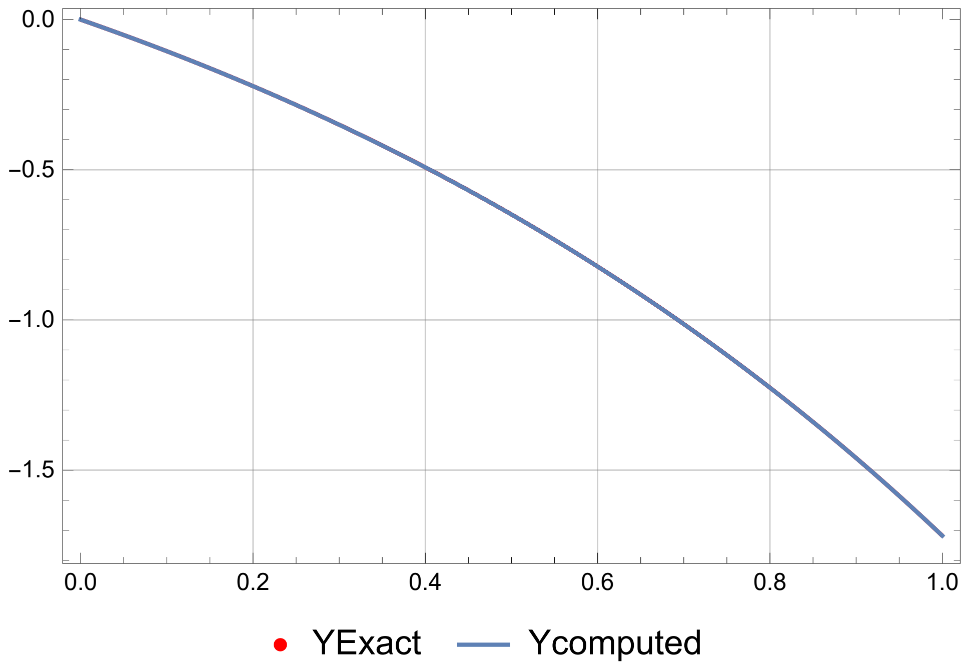

Fig. 3. Graphical Solution of Problem 2 for k=1, p=3 and h=0.01

Problem 3:

The third test problem is the second order IVPs of ODEs
$y'' - x(y')^2 = 0, \quad y(0) = 1, \quad y'(0) = 0.5, \quad h = 0.01,$
whose analytical solution is given as
$y(x) = 1 + \frac{1}{2} In(\frac{2+x}{2-x})$

Table 3. Comparison of Error for Problem 3 (k=1, p=3 and h=0.01)

x-value	y - exact	y - computed	Error	Kayode & Success [22], $k = 2, p = 6, h = 0.01$
0.1	1.005000041667	1.005000041660	6.876 E-12	5.201 E-07
0.2	1.010000333353	1.010000333321	3.251 E-11	1.044 E-06
0.3	1.015001125152	1.015001125065	8.943 E-11	1.707 E-06
0.4	1.020002667307	1.020002667117	1.902 E-10	2.517 E-06
0.5	1.025005210287	1.025005209940	3.474 E-10	3.476 E-06
0.6	1.030009004863	1.030009004289	5.738 E-10	4.591 E-06
0.7	1.035014302180	1.035014301298	8.820 E-10	5.866 E-06
0.8	1.040021353837	1.040021352552	1.285 E-09	7.307 E-06
0.9	1.045030411959	1.045030410164	1.795 E-09	8.920 E-06
1.0	1.050041729278	1.050041726852	2.427 E-09	1.070 E-05

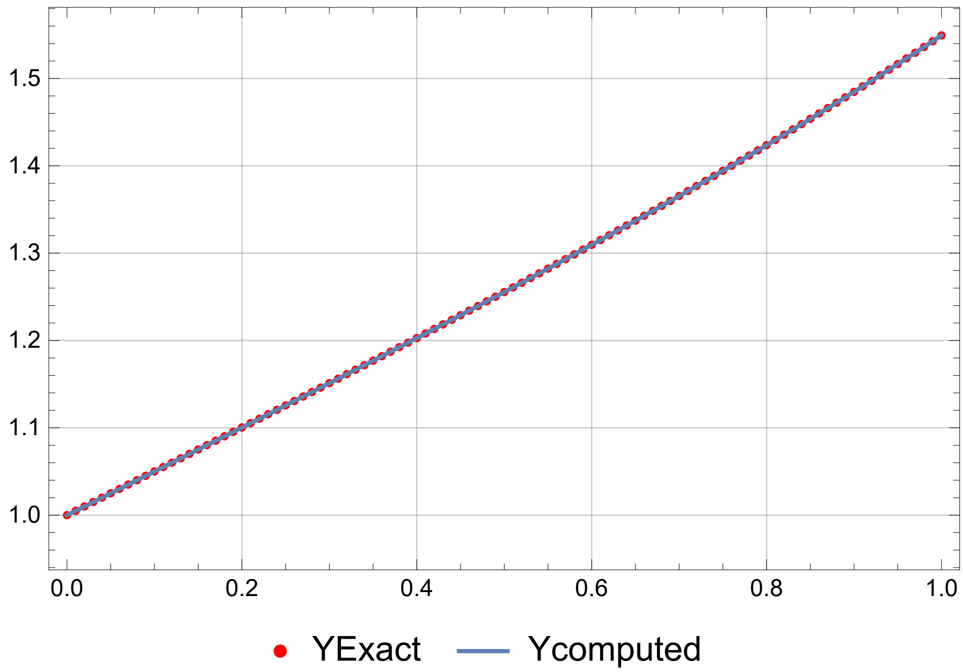

Fig. 4. Graphical Solution of Problem 3 for k=1, p=3 and h=0.01

Problem 4:

Fourth test problem is a third order IVPs of ODEs.
$y''' - y'' + y' - y = 0, \quad y(0) = 1, \quad y'(0) = 0, \quad y''(0) = -1, \quad h = 0.01$
with the exact solution
$y(x) = cosx$

Table 4. Comparison of Error for Problem 4 (k=1, p=3 and h=0.01)

x-value	y -exact	y - computed	Error	Adeyefa [14] $k = 1, p = 7, h = 0.01$
0.1	0.999950000417	0.999950000445	2.831901E-11	1.746452E-07
0.2	0.999800006667	0.999800006892	2.252265E-10	4.159672E-07
0.3	0.999550033749	0.999550034509	7.598979E-10	1.402140E-06
0.4	0.999200106661	0.999200108464	1.803134E-09	3.291541E-06
0.5	0.998750260395	0.998750263922	3.527363E-09	6.348473E-06
0.6	0.998200539935	0.998200546042	6.106603E-09	1.082232E-05
0.7	0.997551000253	0.997551009970	9.716457E-09	1.694540E-05
0.8	0.996801706303	0.996801720837	1.453409E-08	2.493442E-05
0.9	0.995952733012	0.995952753750	2.073822E-08	3.498913E-05
1.0	0.995004165278	0.995004193787	2.850908E-08	1.052497E-04

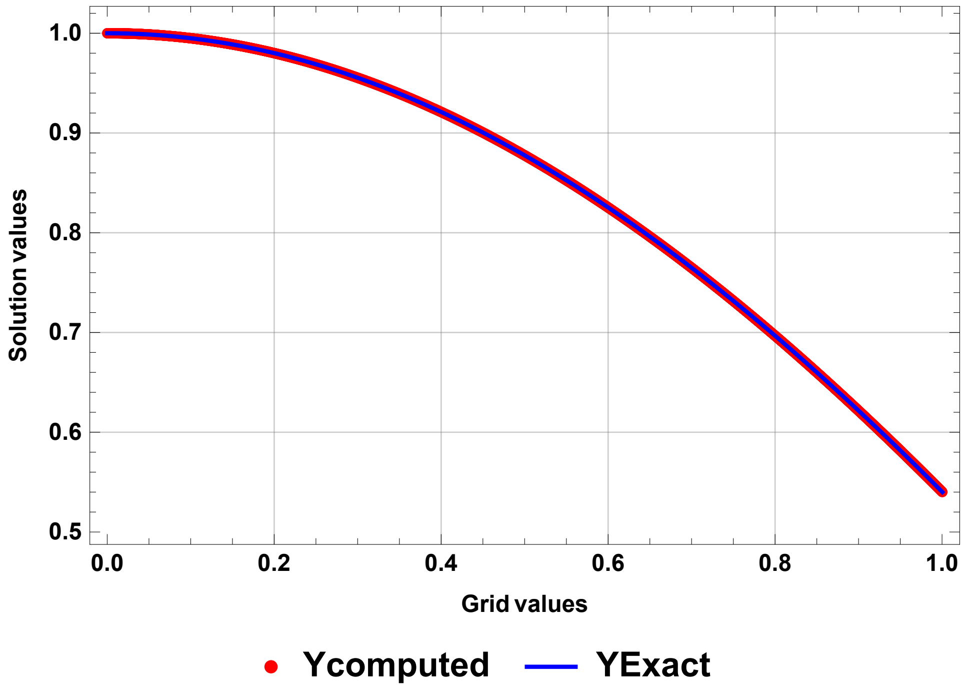

Fig. 5. Graphical Solution of Problem 4 for k=1, p=3 and h=0.01

Problem 5:

The fifth test problem considered is
$y''' = -y', \quad y(0) = 0, \quad y'(0) = 1, \quad y''(0) = 2, \quad h = 0.01$
with the exact solution $y(x) = 2(1 - cosx) + sinx$

Table 5. Comparison of Error for Problem 5 (k=1, p=3 and h=0.01)

x-value	y - exact	y - computed	y - Error	Adeyefa [14] $k = 1, p = 7, h = 0.01$
0.1	0.010099832501	0.010099832416	8.530437E-11	2.33043E-10
0.2	0.020398653360	0.020398652687	6.732482E-10	1.467323E-09
0.3	0.030895432704	0.030895430445	2.259912E-09	4.764563E-09
0.4	0.041589120865	0.041589115526	5.338674E-09	1.123256E-08
0.5	0.052478648481	0.052478638081	1.040011E-08	2.176741E-08
0.6	0.063562926609	0.063562908677	1.793191E-08	3.767493E-08
0.7	0.074840846831	0.074840818412	2.841875 E-08	6.373265E-08
0.8	0.086311281364	0.086311239022	4.234222E-08	9.276724E-08
0.9	0.097973083174	0.097973022993	6.018072 E-08	1.290752E-07
1.0	0.109825086091	0.109825003681	8.240939 E-08	1.757332E-07

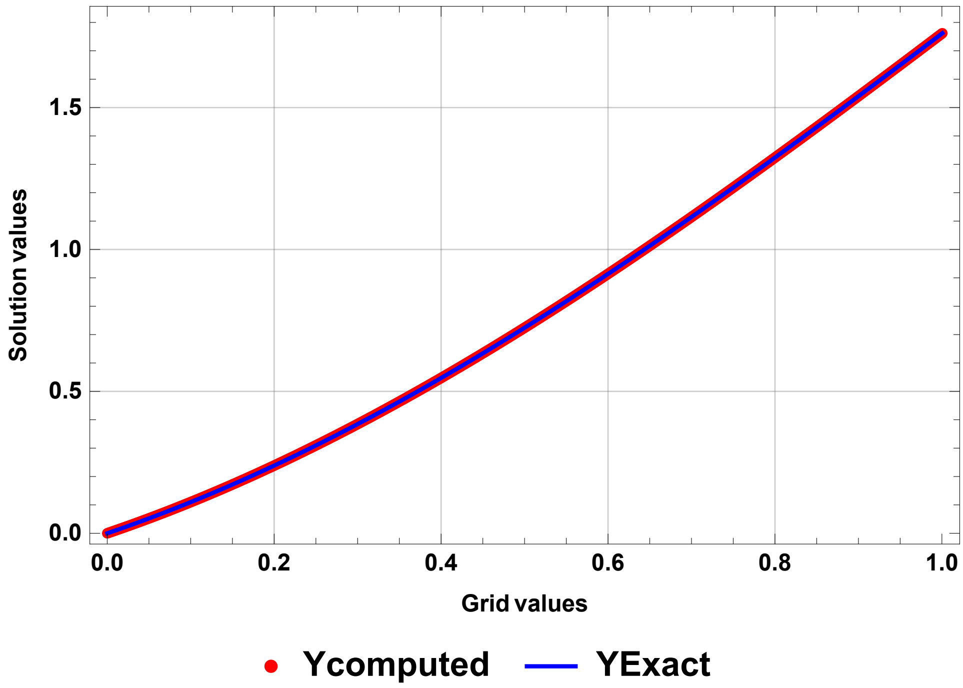

Fig. 6. Graphical Solution of Problem 5 for k=1, p=3 and h=0.01

5 Discussion

Five numerical examples were considered in this paper. Problem 1 is a real-life problem of Artificial Intelligent (AI). The results are presented in Table 1. The method is promising since the computed results agreed with the exact solution up to at least (8) eight decimal places. Problem (2-5) was solved by Ogunware & Ezekiel [18], Kayode & Success [22] and Adeyefa [14] with $h = 0.01$. It was observed that the method performed better than Order Eight of Ogunware & Ezekiel [18], Order Six of Kayode & Success [22], and Order Seven of Adeyefa [14], shown in Tables 2-5. To further analyze the results of the test problems 1-5, the results in Table 1-5 are plotted in Figs. 2-6. It is apparent from the figures that the exact and computed solutions overlap due to the closeness of the solution.

6 Conclusion

This work has provided a suitable and reliable numerical method for solving first, second and third-order initial value problems of ordinary differential equations. A numerical experiment was performed using five test problems. The experiment results indicated that the proposed method is suitable for solving problems described in equation (1). The analysis of the basic properties of the method reveals that the method is convergent of order three, zero-stable and consistent. The derived method is a good choice for handling non-linear problems from the study of real-life situations.

Competing Interests

Authors have declared that no competing interests exist.

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