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Hubble Constant Tension in Terms of Information Approach

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Author's contribution

The sole author designed, analysed, interpreted and prepared the manuscript.

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ABSTRACT

Aims: The purpose of this work is to formulate the theoretically justified information approach to analyze different methods of measuring Hubble's constant, and to verify their advantages and disadvantages.
Place and Duration of Study: Mechanical & Refrigeration Consultation Expert, between June 2019 and November 2019.
Methodology: Due to the fact that any measurement model contains a certain amount of information about the studied object, comparative uncertainty is introduced, by which the least achievable relative uncertainty when measuring the Hubble constant is calculated.
Results: The experimental results of measuring the Hubble constant presented in the scientific literature are analyzed using the proposed information approach.
Conclusion: The information approach can be considered as an additional look at the Hubble constant tension. Most likely, this will help to understand the current situation and identify possible specific ways to solve it.

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1. INTRODUCTION

Scientists are striving to achieve a small amount of uncertainty when measuring physical constants, for at least two reasons. First, with a more accurate knowledge of the numerical values of the constants, we can better understand the universe around us. Secondly, it is not unimportant that the consistency and validity of the basic theories of physics is confirmed precisely thanks to the numerical values of the physical constants calculated using various physical methods.

The value of the Hubble constant characterizes the scale of the length of the Universe and relates the speed of motion of space objects with their distance. In this case, despite the name, the coefficient H_0 is not a constant. Its value has repeatedly changed after the Big Bang. The word "constant" means that at each particular moment in time the value of the coefficient is the same at all points in the universe.

Various methods are used to measure the Hubble constant [1], of which, at the moment, two methods are widely used. The "local approach" is based on studying the behavior of galaxies near our galaxy, the Milky Way, calculating how quickly they move away from each other, and measuring the distances between galaxies in our region of the universe. It is based on the telescopic study of the brightness of distance ladder. In the second method, scientists focus on the study and measurement of cosmic microwave background (CMB), which formed approximately 380,000 years after the Big Bang arose 13.8 billion years ago [2]. The first method gives a value of about 74.3 $km \cdot s^{-1} \cdot Mpc^{-1}$, and the second - about 67 $km \cdot s^{-1} \cdot Mpc^{-1}$, although scientists have declared unprecedented high accuracy achieved by measuring the Hubble constant [3,4]. This mismatch situation is called the Hubble tension. It should be noted that these two values were confirmed by several independent groups, therefore, it can be assumed that the measured values of the Hubble constant H_0 are unlikely to depend on the choice of instrument or the theoretical preferences of a particular team. Only five years ago, it was believed that with a higher accuracy of measurements, this inconsistency could be eliminated. However, the studies, carefully prepared and calculated in detail, made scientists sound the alarm and look for deeper reasons for the existing discrepancy.

The above methods and their results (with data on relative measurement uncertainty and standard uncertainty), which are presented in the scientific literature, will be considered.

The size of the remaining taxonomy indicates that accuracy rather than precision remains the problem of finding the true value of the H_0. The existing discrepancy requires an early explanation and gives rise to the desire of some scientists in search of new ideas: new physics or as yet unrecognized uncertainties. This, in turn, depends on the choice of object-oriented methods, assumptions about other cosmological parameters, and on which data sets are combined in these methods. At the same time, uncertainties are inherent in all measurements and calculations. After taking measurements, scientists find out all the sources of uncertainty that they know with the goal of calculating relative uncertainty. Then, the results of the studies are presented in the form of some integral quantity H_0 and, necessarily, the achieved relative uncertainty is indicated.

Several ideas could be suggested for eliminating the H_0 crisis [5]. First of all, try to identify unaccounted systematic uncertainties in the local method. The fact is that the final result, when using a local remote staircase, carried out in several stages, is very sensitive to a small error in each of them. Although the validity of this idea is very doubtful, given the significant consistency of various local methods. The second idea looks, at the moment, very problematic. It consists in the assumption of the need to create new physics or, at least, to improve the standard ΛCDM model. The third idea is related to the proposal of some physicists to increase the number of neutrino species (more than three). However, the experiments so far confirm only three types of neutrinos. The latter idea, no less exotic, is connected with the possibility of taking into account the energy due to the information contained in the universe. This energy, in its physical content, can be an alternative to dark energy [6,7].

Actually, the very act of the fundamental physical constant measurement already implies the existence of the formulated physical–mathematical model describing the phenomenon under investigation. At the same time, most researchers have focused on data analysis and a calculation of the fundamental physical constant uncertainty value after formulating the mathematical model. But the unavoidable uncertainty existing before the beginning of the experiment or computer simulation, and caused only by the finite number of quantities recorded in the mathematical model of the fundamental physical constant, is generally ignored. Of course, in addition to this uncertainty, the overall uncertainty of the Hubble constant measurement includes the posterior uncertainties related to the internal structure of the model, its subsequent computerization and the testing equipment characteristics: inaccurate input data, inaccurate physical assumptions, the limited accuracy of the solution of integral–differential equations, etc. Detailed definitions of many different sources of uncertainty are given in [8].

In this article, an informational approach is used in the analysis of various measurements of H_0. For this, first of all, the total number of dimensionless criteria contained in the International System of Units is calculated and, accordingly, the entropy value inherent in it. Then, given the fact that a physical and mathematical model has already been formulated for each specific measurement, the amount of information contained in it is calculated. This allows you to determine the relative uncertainty [8], the comparative uncertainty [9] when measuring the Hubble constant and compare those with the experimental values obtained. Since the value of relative uncertainty is realized through a theoretically based information approach, there is the possibility of an objective, independent of any philosophical, scientific and other subjective views of the researcher, assessment of the admissibility of each experimental measurement result. Moreover, when data about new measurements appear, the resulting value of relative uncertainty can be easily updated. The information approach has already been implemented to analyze measurements of other fundamental physical constants [10-13]. This approach allows us to identify the most preferable qualitative set of variables and their optimal number when measuring a physical constant. Which, in turn, is very likely to lead to a reduction in the duration of research, thereby reducing the cost of projects.

2. THE ESSENCE OF THE INFORMATION APPROACH

The main idea of the information approach is as follows. Since all physical laws, natural phenomena in all areas of human activity are represented using, as a rule, variables related to the International System of Units (SI), this system is unique. It does not exist in nature, but it is used according to the developed consensus [14,15].

A distinctive feature of SI is the presence of a finite number of possible physical variables and that is why.

1. SI includes seven (ξ = 7) base quantities: L is length, M is mass, T is time, I is electric current, Θ is thermodynamic temperature, J is luminous intensity, F is the amount of a substance [16].

2. To express the derived quantity q, a combination of the dimensions of the base quantities with different degrees (l, m... f are exponents of the quantities) is used, which, in turn, can only be integers and have a maximum and minimum value [16,17]:

$$\boldsymbol{q} \; \mathring{a} \; \boldsymbol{L}^l \cdot \boldsymbol{M}^m \cdot \boldsymbol{T}^t \cdot \boldsymbol{I}^i \cdot \boldsymbol{\Theta}^\Theta \cdot \boldsymbol{J}^j \cdot \boldsymbol{F}^f \quad (1)$$

$$-3 \le l \le +3,\ -1 \le m \le +1,\ -4 \le t \le +4,\ -2 \le i \le +2 \quad (2)$$
$$-4 \le \theta \le +4,\ -1 \le j \le +1,\ -1 \le f \le +1.$$

$$e_l = 7;\ e_m = 3;\ e_t = 9;\ e_i = 5;\ e_\theta = 9;\ e_j = 3;\ e_f = 3 \quad (3)$$

where, for example, L^{-3} is used in a formula of density, and Θ^4 in the Stefan-Boltzmann law. These examples are given as an explanation and confirmation of the boundaries of the change in the exponents of the main variables in formulas 2 and 3; e_l, e_f are the number of choices of dimensions for each quantity.

Condition (1) is related to the Abelian group and is a very strong restriction. The fact is that if the main value is presented as a correlation function of the selected one-parameter functions, then its use is limited. Exact definitions and application possibilities of an abelian group are formulated in group theory [18]. However, in practice, expression (1) is successfully applied to SI, which exists only in the imagination and is created by human intelligence.

3. In formulating the model, researchers always use both base and derived variables that

correspond to different classes of phenomena (CoP). For example, when measuring the Planck constant, the watt balance method is used, in which four base quantities of SI are involved: *L*-length, *M*- mass, *T*-time, *I*-electric current, i.e., $CoP_{SI} \equiv LMTI$.

4. Taking into account (1) – (3), one can calculate the total number of dimension options of physical quantities $\mathbf{\Psi}° = \prod_l^f e_i$-1

$$\mathbf{\Psi}° = \left(e_l \cdot e_m \cdot e_t \cdot e_i \cdot e_\theta \cdot e_j \cdot e_f - 1\right) = \left(7 \cdot 3 \cdot 9 \cdot 5 \cdot 9 \cdot 3 \cdot 3 - 1\right) = 76{,}544, \quad (4)$$

where "-1" corresponds to the case where all exponents of the base quantities in formula (1) are treated to zero dimension, which corresponds to the case when the quantity has no dimension, $\prod_l^f \boldsymbol{e}_i$ is a product of e_l, … e_f . Equation 4 reflects the situation in which, by common agreement of the scientific community [16, 17], SI has the exact number of variables with a fixed dimension. This fact, until recently, has not been adequately reflected in the scientific literature.

5. If the object under study contains identical elements, then its informational content can be judged only by one part, assuming that the remaining elements are informationally empty. In connection with this postulate, it should be noted that $\mathbf{\Psi}°$ includes both direct and inverse quantities (for example, L^1 is the length, L^{-1} is the running length). Consequently, the number of dimension options can be halved. This means that the total number of variants of the dimensions of physical quantities without inverse quantities is

$$\mathbf{\Psi} = \mathbf{\Psi}°/2 = 38{,}272. \quad (5)$$

6. According to Buckingham π-theorem, which is related to the theory of similarity [19], the number μ_{SI} of possible dimensionless criteria with ξ = 7 base dimensional quantities for SI will be:

$$\boldsymbol{\mu}_{SI} = \mathbf{\Psi} - \xi = 38{,}265. \quad (6)$$

The appropriateness of applying the theory of similarity is explained by the desire to generalize the results obtained for various fields of scientific research. The value of μ_{SI} can only increase with the deepening of knowledge about the world around us. It is necessary to draw the reader's attention to the fact that $\boldsymbol{\mu}_{SI}$ is a mental and artificial system, since it does not exist in physical reality. However, any real physical object can be expressed by this set.

Thus, given that SI is the basis of all the knowledge accumulated by mankind, using the methods of information theory [9], it is possible to calculate the amount of entropy contained in it

$$\boldsymbol{H} = \boldsymbol{k}_b \cdot \ln \boldsymbol{\mu}_{SI}, \quad (7)$$

where $\boldsymbol{H}$ is entropy of SI including $\boldsymbol{\mu}_{SI}$, equally probable accounted quantities, and $\boldsymbol{k}_b$ is the Boltzmann constant.

It should be noted that the statement of the equally probable inclusion of the variable in the model from the point of view of the researcher is justified by the aim of the study: finding the absolute uncertainty $\boldsymbol{\Delta}_{pmm}$ due to the level of detail of the object under study. Indeed, any other distribution of variables gives less information [20-22], which leads to greater model uncertainty compared to the uncertainty calculated with a uniform distribution of variables. In support of the validity of such a statement, the following example can be given: the representation of an electron in the form of a particle or wave. Although two qualitatively different sets of variables are used to describe the motion of an electron, as it turned out, both have the right to life, which led to the concept of electron dualism.

Then, comparing the entropy value inherent in the developed model with the SI entropy corresponding to the maximum number of variables, we obtain the amount of information inherent in the model [23]:

$$\boldsymbol{\Delta A} = \boldsymbol{Q} \cdot \boldsymbol{\Delta H} = \boldsymbol{H}_{pr} - \boldsymbol{H}_{ps}, \quad (8)$$

where $\boldsymbol{\Delta A}$ is the *a priori* information quantity pertaining to the observed object; $\boldsymbol{Q}$ is "the effectiveness of the method of experimental observation, defined as the ratio of the information received to the change in entropy that accompanies the observation." [9] Since the formulation of the model is a thought experiment, distortions are not introduced into the real system, therefore $\boldsymbol{Q}$ = 1; $\boldsymbol{\Delta H}$ is the entropy difference between two cases, and pr is "*a priori*" and ps is "*a posteriori*".

Taking into account (1)-(8), we can prove [23] the following equation, called the $\boldsymbol{\mu}_{SI}$-rule, which declares the relationship between the smallest measurement absolute uncertainty of the main researched variable/function Δ_{pmm}, the interval of its changes S and the number of variables in the model:

$$\Delta_{pmm}/S = [(z'-\beta')/\mu_{SI} + (z''-\beta'')/(z'-\beta')] , \quad (9)$$

where $\varepsilon = \Delta_{pmm}/S$ is the comparative uncertainty [9], z' is the number of physical quantities in the selected CoP and β' is the number of base quantities in the selected CoP, z'' is the number of physical quantities recorded in a mathematical model and β'' is the number of base physical dimensional quantities recorded in a model.

In fact, Δ_{pmm} is an a priori, conceptual, and unrecoverable uncertainty that is inherent in any physical and mathematical model and is independent of the measurement process. It is not a measurement result, but represents an internal property of the model and is determined only by the number of selected quantities and the selected CoP.

The μ_{SI}-rule is applicable to models with both dimensional and dimensionless variables due to the following relations

$$\left(\Delta U/S^*\right)=\left(\Delta U/a\right)/\left(S^*/a\right)=\left(\Delta u/S\right)$$
$$\left(r/R\right)=\left(\Delta U/U\right)/\left(\Delta u/u\right)=\left(\Delta U/U\right)\cdot\left(a/\Delta U\right)\cdot\left(U/a\right)=1 \quad (10)$$

where S and Δu are the dimensionless quantities (respectively, the range of variations and the total absolute uncertainty in determining the dimensionless quantity u); S^* and ΔU are the dimensional quantities (respectively, the range of variations and the total absolute uncertainty in determining the dimensional quantity U); a is the dimensional scale parameter with the same dimension as that of U and S^*; r is the relative uncertainty of the dimensional quantity U; and R is the relative uncertainty of the dimensionless quantity u.

To implement the information approach, first of all, it is necessary to calculate the comparative uncertainty inherent in each CoP corresponding to a specific method of measuring the Hubble constant. To do this, we equate the derivative Δ_{pmm}/S (9) with respect to z'-β' to zero. We can obtain:

$$(\Delta_{pmm}/S)'_{z'-\beta'}=[(z'-\beta')/\mu_{SI}+(z''-\beta'')/(z'-\beta')]'$$
$$=[1/\mu_{SI}-(z''-\beta'')/(z'-\beta')^2], \quad (11)$$

$$[1/\mu_{SI}-(z''-\beta'')/(z'-\beta')^2]=0, \quad (12)$$

$$(z'-\beta')^2/\mu_{SI}=(z''-\beta''), \quad (13)$$

By using (13), one can find the values for the *lowest* achievable comparative uncertainties for different CoP_{SI}; moreover, the values of the comparative uncertainties and the numbers of the chosen variables are *different* for each CoP_{SI}:

1. At measuring the Hubble constant by the brightness of standard candles or the baryon acoustic oscillations (BAO), there are used three base quantities: L, M, *and* T, then $CoP_{SI} = LMT$. The lowest comparative uncertainty ε_{LMT} can be reached at the following conditions:

$$(z'-\beta')_{LMT}=\left(e_l\cdot e_m\cdot e_t-1\right)/2-3=\left(7\cdot 3\cdot 9-1\right)/2-3=91, \quad (14)$$

$$(z''-\beta'')_{LMT}=(z'-\beta')^2_{LMT}/\mu_{SI}=91^2/38,265=0.2164 \quad (15)$$

where "-1" corresponds to the case when all the base quantity exponents are zero; dividing by 2 indicates that there are direct and inverse quantities, e.g., L^1 is the length, L^{-1} is the run length; and 3 corresponds to the three base quantities L, M, T.

According to [9,14 and 15] ε_{LMT} equals:

$$\varepsilon_{LMT}=\left(\Delta u/S\right)_{LMT}=91/38,265+0.2164/91=0.0048 \quad (16)$$

2. At measuring the Hubble constant by the cosmic microwave background, there are used four base quantities: L, M, T, θ, and then $CoP_{SI} = LMT\theta$. The lowest comparative uncertainty $\varepsilon_{LMT\theta}$ can be reached at the following conditions:

$$(z'-\beta')_{LMT\theta}=\left(e_l\cdot e_m\cdot e_t\cdot e_\theta-1\right)/2-4=\left(7\cdot 3\cdot 9\cdot 9-1\right)/2-4=846, \quad (17)$$

$$(z''-\beta'')_{LMT\theta}=(z'-\beta')^2_{LMT\theta}/\mu_{SI}\approx 19 \quad (18)$$

where "-1" corresponds to the case when all the base quantity exponents are zero; dividing by 2 indicates that there are direct and inverse quantities, e.g., L^1 is the length, L^{-1} is the run length; and 3 corresponds to the three base quantities L, M, T, θ.

According to (17) and (18), $\varepsilon_{LMT\theta}$ equals:

$$\varepsilon_{LMT\theta}=846/38,265+19/846\approx 0.0442 \quad (19)$$

Obviously, although the compared methods are designed to measure the same value of H_0, the difference in the formulation of the research problem and the models used, which are qualitatively different from each other, leads to a difference in the values of comparative uncertainties of the mathematical model and to the difference in the requirements for checking the accuracy of the experiment. In other words, the magnitude of the optimal relative uncertainty for each CoP is significantly different (look Chapter 3). In addition, this means that in the framework of the information-oriented approach, the choice of base quantities for measuring the Hubble constant, along with the number of variables taken into account, is crucial in assessing the minimum achievable relative uncertainty. This statement is proved on the basis of a theoretically substantiated method, without using any assumptions.

Analyzing (15), it is necessary to make a preliminary important remark. In the framework of the information approach, when using a model with CoP_{SI} ≡ *LMT*, the required number of dimensionless criteria is less than one. This means that it is impossible not only to achieve, but also to approach the recommended comparative uncertainty and, accordingly, the smallest relative uncertainty. Thus, two methods for measuring the Hubble constant (brightness of standard candles and the baryon acoustic oscillations), in accordance with their internal structure, *initially* cannot be recommended as tools for determining its true value.

Taking into account (16) and (19), the implementation of the information approach is carried out by two methods [24]. The first method dictates the analysis of data on the magnitude of the achievable relative uncertainty at the moment, taking into account the latest measurement results. In this case, the observation range (possible placement interval) of the fundamental physical constant *S* is selected as the difference between the maximum and minimum values of the physical constant measured by various scientific groups over a certain period of recent years. Only with the results of various experiments jointly used to determine H_0 can we talk about a possible random arrangement of the measured value in a certain range. This method is designated as IARU. In the second method, *S* is determined by the limits of the measuring instruments used [9]. This means that as the observation interval in which the expected true value of the measured fundamental physical constant is located, the *standard uncertainty* is chosen in each particular experiment. This method is hereinafter referred to as *IACU*. Examples of detailed explanations of the procedures for using these two methods and the necessary formulas are presented in [10-13].

We apply the μ_{SI}-rule to the analysis of the results of measurements of the Hubble constant.

3. APPLICATIONS OF μ_{SI} FOR HUBBLE CONSTANT MEASUREMENT

The definition of the current "headings" of the best definitions of the Hubble constant can be somewhat subjective. However, most of these lists are likely to include data published during 2009-2019. The results are presented in Tables 1-3 and analyzed from the point of view of the presence of clear achieved values of relative uncertainty and standard uncertainty as a possible interval of the H_0 placement.

3.1 Hubble Constant Measurement by the Brightness of Distance Ladder (BDL)

The reader must bear in mind that measurements made by distance ladder are belong COP_{SI} ≡ *LMT*. Results are introduced in Table 1 [4,25-30].

To apply the stated approach (*IARU*), as the possible measurement interval H_0, there was chosen the difference in its values obtained in two projects: H'_{0min} = 70.6 km·c^{-1}·Mpc^{-1} [26] and H'_{0max} = 74.03 km·c^{-1}·Mpc^{-1} [4]. Then the possible observable range S'_H of the H_0 changes and the average value H'_{0aver} can be represented as follows

$$S'_H = H'_{0max} - H'_{0min} = 3.4 \ (\text{km} \cdot \text{c}^{-1} \cdot \text{Mpc}^{-1}) \quad (20)$$

$$H_{0aver} = (H'_{0max} + H'_{0min})/2 = 72.32 \ (\text{km} \cdot \text{c}^{-1} \cdot \text{Mpc}^{-1}). \quad (21)$$

Applying the *IARU* approach and taking into account (20) and (21), one can calculate the desired values of the absolute Δ_{LMT} and lowest relative r_{LMT} uncertainties

$$\Delta_{LMT} = \varepsilon_{LMT} \cdot S'_H = 0.0442 \cdot 3.4 = 0.0163 \ (\text{km} \cdot \text{c}^{-1} \cdot \text{Mpc}^{-1}) \quad (22)$$

$$r_{LMT} = \Delta_{LMT} / ((H'_{0max} + H'_{0min})/2) = 0.00023. \quad (23)$$

This value (0.00023) is much lower than the declared 0.01 in [4], which confirms the unacceptability of using this method for measuring H_0. The situation is compounded by the fact that the experimental comparative uncertainties calculated in accordance with the

IARU and *IACU* differ significantly from the recommended ε_{LMT} = 0.0048, although progress in achieving higher accuracy has been noted over the past eight years.

In this case, many phenomena, perhaps not significant, secondary, which are characteri zed by specific quantities, are not taken into account.

3.2 Hubble Constant Measurement by CMB

Measurements made by a method of the cosmic microwave background are belong $COP_{SI} \equiv LMT\theta$. Results are introduced in Table 2 [3,32-36].

To apply *IARU*, as the possible measurement interval H_0, there was chosen the difference in its values obtained in two projects: H''_{0min} = 67.4 km·c^{-1}·Mpc^{-1} [36] and H''_{0max} = 71.9 km·c^{-1}·Mpc^{-1} [32]. Then the possible observable range S''_H of the H_0 changes and the average value H''_{0aver} can be represented as follows

$$S''_H = H''_{0max} - H''_{0min} = 4.5 \ (\mathrm{km \cdot c^{-1} \cdot Mpc^{-1}}) \quad (24)$$

$$H''_{0aver} = (H''_{0max} + H''_{0min})/2 = 69.65 \ (\mathrm{km \cdot c^{-1} \cdot Mpc^{-1}}) \quad (25)$$

Applying the *IARU* approach and taking into account (19), (24) and (25), one can calculate the desired values of the absolute $\Delta_{LMT\theta}$ and lowest relative $r_{LMT\theta}$ uncertainties

$$\Delta_{LMT\theta} = \varepsilon_{LMT\theta} \cdot S''_H = 0.0442 \cdot 4.5 = 0.199 \ (\mathrm{km \cdot c^{-1} \cdot Mpc^{-1}}) \quad (26)$$

$$r_{LMT\theta} = \Delta_{LMT\theta} / ((H''_{0max} + H''_{0min})/2) = 0.0029. \quad (27)$$

This value (0.0029) is only 2.4 times less than 0.007 [36]. Judging by the experimental data according to the *IACU* method (relative consistency of the achieved comparative uncertainties), we can make an obvious conclusion that scientists from various research teams carefully analyze the results of other research centers to calculate possible sources of uncertainties. At the same time, using the *IARU* method, it is possible to calculate the achieved comparative uncertainty in each experiment (Table 2). Obviously, there is a large gap between the comparative uncertainty calculated in accordance with the information approach $\varepsilon_{LMT\theta}$ = 0.0442 and the experimental values achieved by measuring H_0 using the CMB method. In the framework of the information approach, the explanation for this almost 2.5-fold difference (0.11 / 0.0442 = 2.5) lies in the unaccounted for relationships between the parameters of the studied objects, which are still hidden for scientists. At this stage, the information approach only indicates the presence of potential additional uncertainties that exist, but have not yet been identified. However, sufficient progress to achieve higher accuracy over the past eight years is obvious. To increase the accuracy of measurements, it is necessary to make specific assumptions about unresolved issues, for example, the nature of dark energy, the global geometry of space and the basic properties of neutrinos (number and mass).

3.3 Hubble Constant Measurement by the Baryonic Acoustic Oscillations (BAO)

To present the application of the information method objectively, we separately analyze the results of measurements of the Hubble constant using BAO, although it is believed that this method is not completely independent of the Planck collaboration measurements (CMB) [1]. Results of measuring H_0 by BAO are introduced in Table 3 [3,31,37,38].

H_0 measurements with BAO belong to $CoP_{SI} \equiv LMT$. To apply *IARU*, there was calculated the possible measurement interval of the H_0 placement as the difference in its values obtained in two projects: H'''_{0min} = 66.98 km ·c^{-1}·Mpc^{-1} [38] and H'''_{0max} = 69.6 km·c^{-1}·Mpc^{-1} [3].

Then the possible observable range S'''_H of the H_0 changes and the average value H'''_{0aver} can be represented as follows

$$S'''_H = H'''_{0max} - H'''_{0min} = 2.62 \ (\mathrm{km \cdot c^{-1} \cdot Mpc^{-1}}) \quad (28)$$

$$H'''_{0aver} = (H'''_{0max} + H'''_{0min})/2 = 68.29 \ (\mathrm{km \cdot c^{-1} \cdot Mpc^{-1}}) \quad (29)$$

Taking into account (28) and (29), one can calculate the desired values of the absolute Δ'''_{LMT} and lowest relative r'''_{LMT} uncertainties

$$\Delta'''_{LMT} = \varepsilon_{LMT} \cdot S'''_H = 0.0048 \cdot 2.62 = 0.0125 \ (\mathrm{km \cdot c^{-1} \cdot Mpc^{-1}}) \quad (30)$$

$$r'''_{LMT} = \Delta'''_{LMT} / ((H'''_{0max} + H'''_{0min})/2) = 0.00018 \quad (31)$$

Table 1. Hubble constant determinations and relative and comparative uncertainties achieved by the brightness of standard candles

Year	Hubble constant	Achieved relative uncertainty	Absolute uncertainty	H_0 possible interval of placing*	Calculated comparative uncertainty	Calculated comparative uncertainty	Ref.
	H_0	r_H	Δ_H	u_H	$\varepsilon_H' = \Delta_H / u_H$ *IACU*	$\varepsilon_H'' = \Delta_H / S_H$ *IARU*	
	$km \cdot c^{-1} \cdot Mpc^{-1}$		$km \cdot c^{-1} \cdot Mpc^{-1}$	$km \cdot c^{-1} \cdot Mpc^{-1}$			
2011	73.8	0.033	2.4	4.8	0.5074	0.71	[25]
2014	70.6	0.047	3.3	6.5	0.5077	0.96	[26]
2016	73.24	0.024	1.74	3.4	0.5118	0.51	[27]
2018	73.48	0.023	1.66	3.2	0.5188	0.48	[28]
2018	72.5	0.032	2.3	4.4	0.5227	0.67	[29]
2018	73.24	0.023	1.7	3.2	0.5313	0.5	[30]
2019	74.03	0.01	0.75	2.2	0.3409	0.22	[4]

** Data are introduced in [2, 4, 31]*

Table 2. Hubble constant determinations and relative and comparative uncertainties achieved by the cosmic microwave background

Year	Hubble constant	Achieved relative uncertainty	Absolute uncertainty	H_0 possible interval of placing*	Calculated comparative uncertainty	Calculated comparative uncertainty	Ref.
	H_0	r_H	Δ_H	u_H	$\varepsilon_H' = \Delta_H / u_H$ *IACU*	$\varepsilon_H'' = \Delta_H / S_H$ *IARU*	
	$km \cdot c^{-1} \cdot Mpc^{-1}$		$km \cdot c^{-1} \cdot Mpc^{-1}$	$km \cdot c^{-1} \cdot Mpc^{-1}$			
2009	71.9	0.038	2.7	5.3	0.5094	0.60	[32]
2011	71.0	0.035	2.5	5.0	0.5000	0.56	[33]
2013	69.32	0.012	0.8	4.4	0.1818	0.18	[34]
2014	69.6	0.010	0.7	1.4	0.5000	0.16	[3]
2016	67.8	0.013	0.9	2.2	0.4091	0.20	[35]
2018	67.4	0.007	0.5	1.0	0.5000	0.11	[36]

** Data are introduced in [2, 4, 31]*

Table 3. Hubble constant determinations and relative and comparative uncertainties achieved by the baryonic acoustic oscillations

Year	Hubble constant	Achieved relative uncertainty	Absolute uncertainty	H_0 possible interval of placing*	Calculated comparative uncertainty	Calculated comparative uncertainty	Ref.
	H_0	r_H	Δ_H	u_H	$\varepsilon_H' = \Delta_H / u_H$ *IACU*	$\varepsilon_H'' = \Delta_H / S_H$ *IARU*	
	km·c^{-1}·Mpc^{-1}		km·c^{-1}·Mpc^{-1}	km·c^{-1}·Mpc^{-1}			
2014	69.6	0.010	0.7	1.4	0.5000	0.27	[3]
2015	68.11	0.013	0.86	1.4	0.6143	0.33	[31]
2018	67.0	0.030	2.0	1.0	2.0000	0.76	[37]
2018	66.98	0.018	1.18	1.6	0.7375	0.45	[38]

** Data are introduced in [2, 4, 31]*

From the perspective of the *IACU* (Table 3), there is uniformity and consistency of the achieved comparative uncertainties, with the exception of [37]. This means that scientists from different research groups are gaining experience from each other in search of unaccounted uncertainties. At the same time, *IARU* calculations indicate an obvious breakthrough in achieving high measurement accuracy over the past four years. However, as part of the information approach, this means that scientists using BAO to calculate H_0 did not take into account some variables and the relationships between them. In other words, they did not identify and reduce some systematic uncertainties to a lower level. In addition, the achieved values of the published relative uncertainties and the comparative uncertainties calculated by *IARU* and *IACU* (Table 3) are very far (two orders of magnitude more!) from the recommended ones (r_{LMT} = 0.00018 and ε_{LMT} = 0.0048). These statements confirm what has been said in the previous Chapter 2: *BAO method cannot be recommended for determining the exact value of the Hubble constant*.

4. RESULTS AND DISCUSSION

The attractiveness of using the brightness of distance ladder or baryonic acoustic vibrations to measure the Hubble constant is due to two main known reasons:

- A standard candle is an object whose luminosity is known. If the brightness of an object is known, then its distance can be calculated by its apparent brightness. To be useful, it must be bright enough to be seen from a great distance;
- The baryonic acoustic vibration of the material group provides a "standard ruler" for the length scale in cosmology.

At the same time, these advantages are leveled from the point of view of the information approach by the insufficient number of variables taken into account, which leads to sharply overestimated values of experimental comparative uncertainty and neglect of possible additional physical relationships between the variables.

It should be noted that the three discussed methods of measuring the Hubble constant have not yet achieved corporate sustainability in establishing its true value. Scientists set goals for continuous improvement of methods, take measures to reduce measurement uncertainty, report on the progress of corporate calculations and increase the number of counted objects. At the same time, each stage of data processing is accompanied by an expert opinion based on the intuition, knowledge and life experience of scientists [39]. However, while tension does not decrease.

A comparison of the considered methods for measuring the Hubble constant is presented in Table 4. From the analysis of the data it follows that there are huge variations between the relative uncertainty calculated in accordance with CoP_{SI} (*IARU*), $\boldsymbol{r}_{SI}$ and the experimental minimum relative uncertainty achieved, $\boldsymbol{r}_{exp}$ from 23 to 100 times for BDL and BAO. This situation is in sharp contradiction with the trend observed when measuring the Boltzmann constant, the Planck constant, and the gravitational constant [13], where the ratio of $\boldsymbol{r}_{exp}/\boldsymbol{r}_{SI}$ is only 0.9-3.0. Only when measuring H_0 with the help of CMB, this ratio is 2.4, which indicates the acceptability and feasibility of using CMB to calculate the Hubble constant value. Since consistency is one of the main requirements in the analysis of results, the current situation requires explanation.

I wonder if this is directly due to the application of the methods themselves, or is this related to the further rejection of data? Perhaps the situation will change for the better if the new method of processing the measurement results H_0 - the Tip of the Branch of the Red Giant – is used [1].

However, there is another reason to explain this situation in the context of the information approach. Already in the process of developing a method for measuring a physical constant, there is an unrecoverable uncertainty called comparative uncertainty, due to the number of variables and a qualitative set of base variables in the model. Its value is not constant and varies depending on the number of registered base variables. Moreover, as mentioned in Chapter 2, the implementation of *LMT* when measuring H_0 is not recommended. This is due to the fact that the achievement of theoretical comparative and relative uncertainties in practice is not possible using this method.

This statement can be confirmed by the following additional argument. The experimental numerical value of the Hubble constant is not just a number used in mathematical processing of the results. Its value, as for other physical constants, is determined by the measurement process, which allows us to establish a relationship between the recorded variables and draw conclusions from the measurement results. In turn, the magnitude of the relative uncertainty is largely determined by the method of measuring H_0 and the experience gained during the experiment. In fact, the experiment is carried out using measuring instruments, depending on the method that identifies specific relationships between the recorded variables and the Hubble constant. At the same time, the use of more accurate measuring instruments allows one to approach only the expected exact value of H_0. This implies the need to introduce the concept of relative uncertainty associated with a set of experimental data for each specific measurement method, which is determined by the class of the phenomenon selected on the basis of the subjective assessment of the research group. Therefore, the conviction of scientists in accounting for all possible sources of uncertainties is far from providing a guarantee of achieving the true value of H_0. The information approach allows us to determine whether the subjectivity of estimating the magnitude of uncertainties (when choosing the class of the phenomenon and the number of variables taken into account) is acceptable when calculating the Hubble constant.

Following the logic of the information approach, it is again necessary to recognize that the method of measuring H_0 using the cosmic microwave background is the most promising, theoretically justified, and implements the most reliable experimental data. This conclusion can be confirmed by calculating the ratio $\varepsilon_{SI}/\boldsymbol{r}_{exp}$ taking into account the data in Table 4

$$\varepsilon_{SI}/r_{exp}: 0.0048/0.018\approx0.3(BAO)<0.0048/0.001\approx4.8(BDL) <0.0442/0.007\approx6\ (CMB). \quad (34)$$

Relation (34) reflects the fact that the best accuracy of measuring the Hubble constant can be achieved for the class of phenomena with a large number of base quantities.

Summing up all of the above, it should be noted that modern physics has achieved significant success in explaining the functioning of the material world, but when it comes to the Hubble constant, in the last decade, scientists still do not have a clear explanation of the existing discrepancy in calculating its magnitude. This, in the long run, may lead to fundamental changes in the scientific worldview. Therefore, the information approach can be considered as an additional look at the existing problem. This, most likely, will help to understand the current situation and determine specific ways to solve it.

Perhaps the methods considered are currently limited to taxonomy; in other words, the subject is limited by accuracy, not precision, and that close attention to the underrated taxonomy will bring values closer together over the next few years. At the same time, great care must be taken when formulating forecasts of improving the accuracy of the Hubble constant measurement. The fact is that, with an increase in the number of observed space objects, according to the majority of astronomers using various methods of calculating H_0, the absolute (ideal) statistical stability of the parameters and characteristics of any physical phenomena (real events, quantities, processes and fields) will be observed. However, as was proved [39], the key role in limiting accuracy is played by the non-ideal nature of statistical stability (statistical predictability), which manifests itself in the absence of convergence (failure) of statistical estimates. At small time, spatial or space-time intervals of observation, an increase in the volume of statistical data leads to a decrease in the level of fluctuations in statistical estimates, which creates the illusion of ideal statistical stability. But, starting with a certain critical amount of data, the decrease in the level of fluctuations ceases. A further increase in the number of data either practically does not affect the level of fluctuations in the estimates, or even leads to their growth.

Many readers may have concerns that a methodology using an information approach does not have sufficient justification for use in astronomy. Many of them may argue that the presented results are not sufficient evidence and contradict the existing vision of scientists when analyzing data on the measurement of H_0. Nevertheless, the relationship between the accuracy of the Hubble constant measurement and the choice of the class of phenomena, as well as the number of variables taken into account, cannot be ruled out.

It may seem that the author criticizes all the methods. Not because he considers them extremely bad - on the contrary, the author is convinced that they are based on important physical ideas and an excellently developed mathematical apparatus for processing experimental results when measuring the Hubble constant. The tools used have shown the high efficiency achieved in recent years. That is why, thinking about the pitfalls that await us in the

Table 4. Summarized data

Variable /Method	BDL	CMB	BAO
CoP	*LMT*	*LMTθ*	*LMT*
Comparative uncertainty according to CoP_{SI}, ε_{SI}	0.0048	0.0442	0.0048
$S_H = H_{0max} – H_{0min}$, km c^{-1} Mpc^{-1}	3.4	4.5	2.6
Relative uncertainty according to CoP_{SI} (*IARU*), r_{SI}	0.00023	0.0029	0.00018
Achieved experimental lowest relative uncertainty, r_{exp}	0.001	0.007	0.018
Ratio of r_{exp} / r_{SI}	23	2.4	100

study of the universe, it is important to identify the weaknesses of the H_0 measurement methods and find out how to change and improve them.

5. CONCLUSION

The information-oriented approach is theoretically justified and founded on the reliable notions of observation and repeatability for calculating the relative uncertainty in measuring the value of H_0. It is easy to use, does not require consistency checking, and works well without using any statistical and expert assumptions based on personal philosophical inclinations [40]. A detailed description of the data and processing procedures do not require significant time, high quality personnel and a significant budget. Its use as a universal tool is demonstrated by the example of processing the data of the simulated interval and the real values of the Hubble constant presented in scientific studies for the period from 2009 to 2019.

Based on the presented analysis, the author considers it necessary to formulate the following theoretical provisions:

- The International System of Units (SI) does not exist in nature, it is an abstract mathematical concept or it can be called the “free creation of the human mind” [41]. At the same time, SI is the basis of intellectual knowledge that scientists use to describe the world around us at present;

- The ability to accurately describe natural phenomena and any technological processes is limited by a fundamental limit, which does not depend on the characteristics of the measuring process (improving tools, measurement methods or computerization of the model), is an internal property of the model and is calculated in accordance with the chosen class of the phenomenon and the number of quantities taken into account.

- Any models based on the selection of a small number of base quantities, for example, MT, LMT, LMJ, and so on, are not recommended for measuring both the Hubble constant and other physical constants. This statement is not only of academic interest. Neglect and elimination of possible hidden connections of observed cosmological objects impede a clear understanding of the advantages and disadvantages of various methods of measuring H_0 and lead to logical conflicts.

- In the framework of the information-oriented approach, the choice of base variables for measuring the Hubble constant, along with the number of variables taken into account, is crucial in assessing the minimum achievable relative uncertainty. This statement is proved on the basis of a theoretically substantiated method, without using any assumptions.

Thus, the proposed information method is able to make several verified forecasts, in particular, on the preference for the cosmic microwave background when measuring the Hubble constant.

DISCLAIMER

The products used for this research are commonly and predominantly use products in our area of research and country. There is absolutely no conflict of interest between the author and producers of the products because i do not intend to use these products as an avenue for any litigation but for the advancement of knowledge. Also, the research was not funded by the producing company rather it was funded by personal efforts of the author

ETHICAL APPROVAL

1. I, the alone corresponding author, am authorized to submit this manuscript.
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Author has declared that no competing interests exist.

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