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Five Points Mono Hybrid Point Linear Multistep Method for Solving Nth Order Ordinary Differential Equations Using Power Series Function

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Authors' contributions

This work was carried out in cooperation between both authors. Author ABF designed the study, wrote the concept of the article, taken care of formatting and provided finishing touch to this manuscript. Author EOO managed the experimental process and wrote the first draft of the manuscript and also managed the experimental process, along with the literature searches. Author ABF also managed the analyses of the study. Both authors read and approved the final manuscript.

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Abstract

In this paper, a class of five point's mono hybrid linear multistep method for solving nth order ordinary differential equations has been developed. In the derivation, Power series approximation was used as basic function due to its accuracy and simplicity. Collocation and Interpolation was adopted to generate the system of equations which was solved for the unknown parameters in the approximation solution of the problem. The derived schemes were implemented both in block and Pc method of the same order of accuracy. The schemes were tested on some numerical test problem ranging from linear to non-linear and problem from ship dynamics. The results show that block method performs better in term of accuracy. The basic properties of the methods show that they are zero stable, consistence and convergence.

Keywords: Mono hybrid point; Nth order; application problem; LMM; zero stable; block method.

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1 Introduction

Differential equation plays an important role in the modeling of physical problems arising from almost every discipline of study. In science and engineering, usually mathematical models are developed to help in the understanding of physical phenomena. These models often yield equations that contain some derivatives of an unknown function of one or several variables. Such differential equations are called Differential equation. An example of such models is the radioactive decay model given by the equation:

$$\frac{dA}{dt} = -kA, \; k > 0 \tag{1}$$

where A is the unknown amount of radioactive substance present at time t and k is the proportionality constant. Equation (1) is called ordinary differential equation (ODE) because the dependent variable A contains only one independent variable t.

Differential equations arise in many areas of science and technology, specifically whenever a deterministic relation involving some continuously varying quantities (modeled by functions) and their rates of change in space and/or time (expressed as derivatives) is known or postulated. This is illustrated in classical mechanics, where the motion of a body is described by its position and velocity as the time varies. Newton's laws allow one to relate the position, velocity, acceleration and various forces acting on the body and state this relation as a differential equation for the unknown position of the body as a function of time.

Consider the nth order initial value problems of ordinary differential equation of the form

$$y^n = f\left(x, y, y^{n-3}, y^{n-2}, y^{n-1}\right), y\left(x_0\right) = \tau_0, y^{n-3}\left(x_0\right) = \tau_1, y^{n-2}\left(x_0\right) = \tau_2, y^{n-1}\left(x_0\right) = \tau_3 \tag{2}$$

Suppose n is specifically choose as 4, then (2) becomes

$$y^{iv} = f\left(x, y, y', y'', y'''\right), y\left(x_0\right) = \tau_0, y'\left(x_0\right) = \tau_1, y''\left(x_0\right) = \tau_2, y'''\left(x_0\right) = \tau_3 \tag{3}$$

on the interval [a,b] has given rise to two major discrete variables methods namely;

One-step or Single step method and Multistep methods especially the Linear Multistep method. Example of one step method include the Euler's methods, the Runge-Kutta methods, The theta methods, the Taylor's series, Obrechoff method etc. these methods are only suitable for the solution of first order initial value problems of ordinary differential equations because of their very low order of accuracy.

A linear multistep method includes Adams-Bashforth, Adams-moulton method, Backward differential method and Numerov method. These methods give high order of accuracy and are suitable for direct solution of (2) without the necessary reducing it to an equivalent system of first order differential equations. This research work is centered on the development five points mono hybrid point with the step length of four for solving fourth order ordinary differential equation directly without using a conventional method.

1.1 Preliminaries

1.1.1 Linear multistep method

The general form of a linear k-step multistep method is

$$y(x)=\sum_{i=0}^{k}\alpha_i y_{n+i}+h^n\sum_{i=0}^{k}\beta_i f_{n+i} \tag{4}$$

where $y(x)$ is the numerical solution of the initial value problem, while α_i and β_i are constants and n is the order of the differential equation.

Linear multistep method can be divided into two, as follow

i. **Implicit linear multistep method**: The linear multistep method is said to be implicit if $\beta_k \neq 0$ in (4).

ii. **Explicit linear multistep method**: The linear multistep method is said to be explicit if $\beta_k = 0$ in (4).

1.1.2 Collocation method and interpolation

Collocation is the evaluation of the differential systems at some specified/selected grid point while Interpolation is the evaluation of the approximate solution at some selected/specified grid points.

1.1.3 Initial value problem (IVP)

An initial value problem is an ordinary differential equation with a specified value known as the initial condition of the unknown function at a given point in the domain of the solution.

This can also be defined as an Ordinary differential equation with a set of initial value problem.

The general form of nth order initial value problem is

$$y^n = f(x,y,y'........y^{n-1}),\ y(x_i)=y_i,\ i=0(1)n-1 \tag{5}$$

2 Literature Review

Conventionally, fourth order ordinary differential equations is usually reduced to system of first order differential equations before an approximate method is applied to solve it. This reduction approach has been extensively discussed by several authors see ([1,2,3,4,5,6,7,8,9,10,11]). It was reported that due to the dimension of the resulting system of first order ordinary differential equations, the approach involves large human efforts. According to ([12,13,11,14]), the major drawback of this approach of solution is that writing computer programs for these methods is often complicated especially when subroutines are incorporated to supply starting values required for the methods. The consequences of this are that longer time and human efforts are involved. Many prominent authors including ([8,10,15,16,13,17,18]) etc. have developed methods for the direct solution of (2) without reducing it to systems of first order ordinary differential equations. [17], independently proposed multi-derivative linear multistep methods and implemented in a predictor-corrector mode using Taylor series algorithm to supply the starting values. Although this kind of implementation yielded good accuracy, the procedure is more costly to implement because predictor-corrector subroutines involved are very complicated, since they require special techniques to supply the starting values for varying the step-size which leads to longer computer time and more human effort. This is extensively discussed by ([10,16,13,14] and [17]). [17] developed linear multi step method for the solution of general and special third order ordinary differential equations. The need to improve on the predictor-corrector method so as to obtain another better method became important to researchers in this area. Thus, a continuous linear multi step method that computes the discrete methods at more than one point simultaneously was developed. This is what is now referred to as Block method.

Considerable attention is now being paid to this method of solving initial value problems of higher order ordinary differential equations of the form:

$$f\left(x, y, y', y'', \ldots, y^{(n-1)}, y^{n}\right) = 0 \tag{6}$$

where n is the order of the ordinary differential equation, x is the independent variable, y is the dependent variable and $y', y'', \ldots, y^{(n-1)}$ are the derivatives of the dependent variable with respect to independent variable. The researchers that have worked in this area include ([1,2,12,13,14,17,19,20,21,18] and [22]). [14] presented numerous methods for direct solution of fourth order ordinary differential equations for solving initial value problems, it was observed that the methods are consistence and zero stable.

This research paper is motivated by the need to address the setbacks associated with the existing methods and also to serve an alternative method by developing a new method which can solve directly fourth order ordinary differential equation without the need to reduce it to the system of first order ordinary differential equation.

3 Mathematical Formulation of the Method

Consider power series as an approximate solution to the general fourth order problems.

$$y^{iv} = f\left(x, y, y', y'', y'''\right), y\left(x_0\right) = \tau_0, y'\left(x_0\right) = \tau_1, y''\left(x_0\right) = \tau_2, y'''\left(x_0\right) = \tau_3, \tag{7}$$

to be of the form

$$y(x) = \sum_{j=0}^{9} a_j x^j \tag{8}$$

Where a_j are parameters to be determined

The fourth derivatives of (8) is;

$$y^{iv}(x) = \sum_{j=0}^{9} j(j-3)(j-2)(j-1)a_j x^{j-4} \tag{9}$$

Substituting (8) into (9) we obtain

$$\sum_{j=0}^{9} j(j-3)(j-2)(j-1)a_j x^{j-4} = f(x, y, y^{'}, y^{''}, y^{'''}) \tag{10}$$

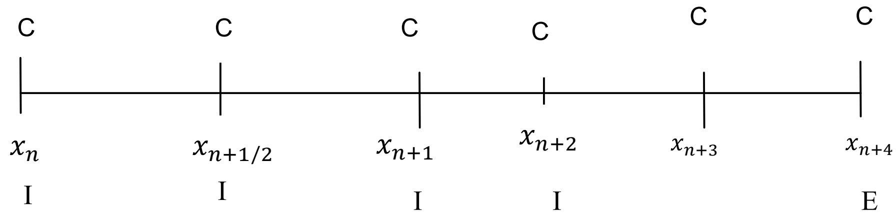

Fig. 1. Diagrammatic representation of the scheme

Where $C=$ Points of collocation, $I=$ Points of interpolation and $E=$ Point of evaluation

Collocate equation (9) at $x=x_{n+i}, i=0,\frac{1}{2},1,2,3,4$, Interpolate (8) at $x=x_{n+i}, i=0,\frac{1}{2},1,2$ and evaluating at the end point $x=x_{n+i}, i=3,4$. This can be represented in matrix as follows

$$\begin{bmatrix} 1 & x_n & x_n^2 & x_n^3 & x_n^4 & x_n^5 & x_n^6 & x_n^7 & x_n^8 & x_n^9 \\ 1 & x_{n+\frac{1}{2}} & x_{n+\frac{1}{2}}^2 & x_{n+\frac{1}{2}}^3 & x_{n+\frac{1}{2}}^4 & x_{n+\frac{1}{2}}^5 & x_{n+\frac{1}{2}}^6 & x_{n+\frac{1}{2}}^7 & x_{n+\frac{1}{2}}^8 & x_{n+\frac{1}{2}}^9 \\ 1 & x_{n+1} & x_{n+1}^2 & x_{n+1}^3 & x_{n+1}^4 & x_{n+1}^5 & x_{n+1}^6 & x_{n+1}^7 & x_{n+1}^8 & x_{n+1}^9 \\ 1 & x_{n+2} & x_{n+2}^2 & x_{n+2}^3 & x_{n+2}^4 & x_{n+2}^5 & x_{n+2}^6 & x_{n+2}^7 & x_{n+2}^8 & x_{n+2}^9 \\ 0 & 0 & 0 & 0 & 24 & 120x_n^2 & 360x_n^3 & 840x_n^4 & 1680x_n^5 & 3024x_n^6 \\ 0 & 0 & 0 & 0 & 24 & 120x_{n+\frac{1}{2}}^2 & 360x_{n+\frac{1}{2}}^3 & 840x_{n+\frac{1}{2}}^4 & 1680x_{n+\frac{1}{2}}^5 & 3024x_{n+\frac{1}{2}}^6 \\ 0 & 0 & 0 & 0 & 24 & 120x_{n+1}^2 & 360x_{n+1}^3 & 840x_{n+1}^4 & 1680x_{n+1}^5 & 3024x_{n+1}^6 \\ 0 & 0 & 0 & 0 & 24 & 120x_{n+2}^2 & 360x_{n+2}^2 & 840x_{n+2}^4 & 1680x_{n+2}^5 & 3024x_{n+2}^6 \\ 0 & 0 & 0 & 0 & 24 & 120x_{n+3} & 360x_{n+3}^2 & 840x_{n+3}^4 & 1680x_{n+3}^5 & 3024x_{n+3}^6 \\ 0 & 0 & 0 & 0 & 24 & 120x_{n+4} & 360x_{n+4}^2 & 840x_{n+4}^3 & 1680x_{n+4}^4 & 3024x_{n+4}^5 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \\ a_6 \\ a_7 \\ a_8 \\ a_9 \end{bmatrix} = \begin{bmatrix} y_n \\ y_{n+\frac{1}{2}} \\ y_{n+1} \\ y_{n+2} \\ f_n \\ f_{n+1/2} \\ f_{n+1} \\ f_{n+2} \\ f_{n+3} \\ f_{n+4} \end{bmatrix} \quad (11)$$

Solving for $a_j, j=0(1)9$ in (11) using Gaussian elimination method and substituting the values of $a_j, j=0(1)9$ into (8) gives a linear multistep method with continuous coefficients in the form:

$$y(x)=\sum_{j=0}^{k-1}\alpha_j(x)y_{n+j}+h^4\left(\sum_{j=0}^{k}\beta_j(x)f_{n+j}+\beta_v(x)f_{n+v}\right) \quad (12)$$

where $y(x)$ is the numerical solution of the initial value problem and $v=\frac{1}{2}$. α_j and β_j are constants. $f_{n+j}=f(x_{n+j},y_{n+j},y'_{n+j},y''_{n+j},y'''_{n+j})$

Using the transformation $t=\frac{x-x_{n+3}}{h}, \frac{dt}{dx}=\frac{1}{h}$

The coefficients of y_{n+j} and f_{n+j} are obtained as:

$$\alpha_0(t) = \frac{-1}{2}(10+19t+11t^2+2t^3)$$
$$\alpha_{1/2}(t) = \frac{8}{3}(6+11t+6t^2+t^3)$$
$$\alpha_1(t) = -(15+26t+13t^2+2t^3)$$
$$\alpha_2(t) = \frac{1}{6}(30+30t+15t^2+2t^3)$$
$$\beta_0(t) = \frac{-h^4}{725760}\left[-11970-23493t-10872t^2+2269t^3-2520t^5-756t^6+288t^7+168t^8+20t^9\right] \quad (13)$$
$$\beta_{1/2}(t) = \frac{h^4}{3175200}\left[-127890-272481t-117330t^2+59005t^3-48384t^5-13440t^6+5760t^7+2880t^8+320t^9\right]$$
$$\beta_1(t) = \frac{-h^4}{1451520}\left[-663390-1324791t-782574t^2-100645t^3-30240t^5-7392t^6+3744t^7+1584t^8+160t^9\right]$$
$$\beta_2(t) = \frac{h^4}{1451520}\left[291690+786621t+770898t^2+301543t^3-30240t^5-2352t^6+3168t^7+936t^8+80t^9\right]$$
$$\beta_3(t) = \frac{-h^4}{725760}\left[76230+53067t-258570t^2-461215t^3-302400t^4-74592t^5+10080t^6+10080t^7+2160t^8+160t^9\right]$$
$$\beta_4(t) = \frac{h^4}{10160640}\left[13230+20007t-5694t^2-19507t^3+15120t^5+11256t^6+3744t^7+612t^8+40t^9\right]$$

Evaluating (13) at the non-interpolation points i.e t=4 and t = 3 gives the discrete schemes below

$$y_{n+4}-14y_{n+2}+56y_{n+1}-64y_{n+1/2}+21y_n = \frac{h^4}{2880}[186f_n-464f_{n+1/2}+5761f_{n+1}+4211f_{n+2}+375f_{n+3}+11f_{n+4}] \quad (14)$$
$$y_{n+3}-5y_{n+2}+15y_{n+1}-16y_{n+1/2}+5y_n = \frac{h^4}{11520}[190f_n-464f_{n+1/2}+5265f_{n+1}+2315f_{n+2}-121f_{n+3}] \quad (15)$$

Finding the first derivative of (13) gives:

$$\alpha_0'(t) = \frac{-1}{2h}(19+22t+6t^2)$$
$$\alpha_{\frac{1}{2}}'(t) = \frac{8}{3h}(11+12t+3t^2)$$
$$\alpha_1'(t) = \frac{-1}{h}(26+26t+6t^2)$$
$$\alpha_2'(t) = \frac{1}{6h}(37+30t+6t^2)$$
$$\beta_0'(t) = \frac{-h^3}{725760}\left[-23493-21744t+6807t^2-12600t^4-4536t^5+2016t^6+1344t^7+180t^8\right]$$
$$\beta_{1/2}'(t) = \frac{h^3}{3175200}\left[-272481-234660t+177015t^2-241920t^4-80640t^5+40320t^6+23040t^7+2880t^8\right] \quad (16)$$
$$\beta_1'(t) = \frac{h^3}{1451520}\left[-1324791-1565148t-301935t^2-151200t^4-44352t^5+26208t^6+12672t^7+1440t^8\right]$$

$$\beta_2'(t)=\frac{h^3}{1451520}\left[786621+1541797t+904629t^2-151200t^4-14112t^5+22176t^6+74688t^7+720t^8\right]$$
$$\beta_3'(t)=\frac{-h^3}{7257600}\left[53067-517140t-1383645t^2-1209600t^3-372960t^4+60480t^5+70560t^6+17280t^7+1440t^8\right]$$
$$\beta_4'(t)=\frac{h^3}{10160640}\left[20007-11388t-58521t^2+75600t^4+67536t^5+26208t^6+4896t^7+360t^8\right]$$

Finding the second derivative of (13) gives:

$$\alpha_0''(t)=\frac{-1}{2h^2}(22+12t)$$
$$\alpha_{1/2}''(t)=\frac{8}{3h^2}(12+6t)$$
$$\alpha_1''(t)=\frac{-1}{h^2}(26+12t)$$
$$\alpha_2''(t)=\frac{1}{6h^2}(30+12t)$$

$$\beta_0''(t)=\frac{-h^2}{725760}\left[-21744+13614t-50400t^3-22680t^4+12096t^5+9408t^6+1440t^7\right] \quad (17)$$
$$\beta_{1/2}''(t)=\frac{h^2}{3175200}\left[-234660+354030t-967680t^3-403200t^4+241920t^5+161280t^6+23040t^7\right]$$
$$\beta_1''(t)=\frac{h^2}{1451520}\left[-1565148-603870t-604800t^3-221760t^4+157248t^5+88704t^6+11520t^7\right]$$
$$\beta_2''(t)=\frac{h^2}{1451520}\left[1541796+1809258t-604800t^3-70560t^4+133056t^5+522816t^6+5760t^7\right]$$
$$\beta_3''(t)=\frac{-h^2}{725760}\left[-517140-2767290t-3628800t^2-1491840t^3+302400t^4+423360t^5+120960t^6+11520t^7\right]$$
$$\beta_4''(t)=\frac{h^2}{10160640}\left[-11388-117042t+302400t^3+337680t^4+157248t^5+34272t^6+2880t^7\right]$$

finding the third derivative of (13) gives:

$$\alpha_0'''(t)=\frac{-6}{h^3}$$
$$\alpha_{1/2}'''(t)=\frac{16}{h^3}$$
$$\alpha_1'''(t)=\frac{-12}{h^3}$$
$$\alpha_2'''(t)=\frac{2}{h^3}$$
$$\beta_0'''(t)=\frac{-h}{725260}\left[13614-151200t^2-90720t^3+60480t^4+56448t^5+10080t^6\right] \quad (18)$$
$$\beta_{1/2}'''(t)=\frac{h}{3175200}\left[354030-2903040t^2-1612800t^3+1209600t^4+967680t^5+161280t^6\right]$$
$$\beta_1'''(t)=\frac{h}{1451520}\left[-603870-1814400t^2-887040t^3+786240t^4+532225t^5+80640t^6\right]$$
$$\beta_2'''(t)=\frac{h}{1451520}\left[1809258-1814400t^2-282240t^3+665280t^4+3136896t^5+40320t^6\right]$$
$$\beta_3'''(t)=\frac{-h}{725260}\left[-2767290-7257600t^2+1209600t^3+2116800t^4+725760t^5+80640t^6\right]$$
$$\beta_4'''(t)=\frac{h}{10160640}\left[-117042+907200t^2+1350720t^3+786240t^4+205632t^5+20160t^6\right]$$

Evaluating (13) at t= 3, 4, the first, second and third derivatives of (13) at the entire grid and off grid points i.e at t=0, 1/2, 1, 2, 3,4 gives the discrete schemes which are combined together to form the required methods below

$$\begin{bmatrix}1&0&0&0&0\\0&1&0&0&0\\0&0&1&0&0\\0&0&0&1&0\\0&0&0&0&1\end{bmatrix}\begin{bmatrix}y_{n+1/2}\\y_{n+1}\\y_{n+2}\\y_{n+3}\\y_{n+4}\end{bmatrix}=\begin{bmatrix}0&0&0&0&1\\0&0&0&0&1\\0&0&0&0&1\\0&0&0&0&1\\0&0&0&0&1\end{bmatrix}\begin{bmatrix}y_{n-1/2}\\y_{n-1}\\y_{n-2}\\y_{n-3}\\y_n\end{bmatrix}+h\begin{bmatrix}0&0&0&0&\frac{1}{2}\\0&0&0&0&1\\0&0&0&0&2\\0&0&0&0&3\\0&0&0&0&4\end{bmatrix}\begin{bmatrix}y'_{n-1/2}\\y'_{n-1}\\y'_{n-2}\\y'_{n-3}\\y'_n\end{bmatrix}$$

$$+h^2\begin{bmatrix}0&0&0&0&\frac{1}{8}\\0&0&0&0&\frac{1}{2}\\0&0&0&0&2\\0&0&0&0&\frac{9}{2}\\0&0&0&0&8\end{bmatrix}\begin{bmatrix}y''_{n-1/2}\\y''_{n-1}\\y''_{n-2}\\y''_{n-3}\\y''_n\end{bmatrix}+h^3\begin{bmatrix}0&0&0&0&\frac{1}{48}\\0&0&0&0&\frac{1}{6}\\0&0&0&0&\frac{4}{3}\\0&0&0&0&\frac{9}{2}\\0&0&0&0&\frac{32}{2}\end{bmatrix}\begin{bmatrix}y'''_{n-1/2}\\y'''_{n-1}\\y'''_{n-2}\\y'''_{n-3}\\y'''_n\end{bmatrix} \tag{19}$$

$$+h^4\begin{bmatrix}\frac{4847}{3628800}&\frac{-197}{331776}&\frac{319}{2322432}&\frac{-2077}{58060800}&\frac{11}{2322432}\\\frac{2938}{99225}&\frac{-853}{90720}&\frac{391}{181440}&\frac{-253}{453600}&\frac{187}{2540160}\\\frac{45056}{99225}&\frac{8}{2835}&\frac{74}{2835}&\frac{-88}{14175}&\frac{16}{19845}\\\frac{2214}{1225}&\frac{135}{224}&\frac{135}{448}&\frac{-27}{800}&\frac{27}{6272}\\\frac{65536}{14175}&\frac{1024}{405}&\frac{4864}{2835}&\frac{1024}{14175}&\frac{32}{2835}\end{bmatrix}\begin{bmatrix}f_{n+1/2}\\f_{n+1}\\f_{n+2}\\f_{n+3}\\f_{n+4}\end{bmatrix}+h^4\begin{bmatrix}0&0&0&0&\frac{2039}{1161216}\\0&0&0&0&\frac{7181}{362880}\\0&0&0&0&\frac{536}{2835}\\0&0&0&0&\frac{621}{896}\\0&0&0&0&\frac{4864}{2835}\end{bmatrix}\begin{bmatrix}f_{n-1/2}\\f_{n-1}\\f_{n-2}\\f_{n-3}\\f_n\end{bmatrix}$$

Writing out (19) explicitly, we have

$$y_{n+\frac{1}{2}}=y_n+\frac{1}{2}hy'_n+\frac{1}{8}h^2y''_n+\frac{1}{48}h^3y'''+h^4\left(\begin{array}{l}\frac{2039}{1161216}f_n+\frac{4847}{3628800}f_{n+1/2}-\frac{197}{331776}f_{n+1}+\\\frac{319}{2322432}f_{n+2}-\frac{2077}{58060800}f_{n+3}+\frac{11}{2322432}f_{n+4}\end{array}\right) \tag{20}$$

$$y_{n+1}=y_n+hy'_n+\frac{1}{2}h^2y''_n+\frac{1}{6}h^3y'''+h^4\left(\begin{array}{l}\frac{7181}{362880}f_n+\frac{2938}{99225}f_{n+1/2}-\frac{853}{90720}f_{n+1}+\frac{319}{181440}f_{n+2}\\-\frac{253}{453600}f_{n+3}+\frac{187}{2540160}f_{n+4}\end{array}\right) \tag{21}$$

$$y_{n+2}=y_n+2hy'_n+2h^2y''_n+\frac{4}{3}h^3y'''+h^4\left(\begin{array}{l}\frac{536}{2835}f_n+\frac{45056}{99225}f_{n+1/2}+\frac{8}{2835}f_{n+1}+\frac{74}{2835}f_{n+2}-\\\frac{88}{14175}f_{n+3}+\frac{16}{19845}f_{n+4}\end{array}\right) \tag{22}$$

$$y_{n+3} = y_n + 3hy'_n + \frac{9}{2}h^2 y''_n + \frac{9}{2}h^3 y''' + h^4 \left(\begin{array}{l} \frac{621}{896} f_n + \frac{2214}{1225} f_{n+1/2} + \frac{135}{224} f_{n+1} + \frac{135}{448} f_{n+2} - \\ \frac{27}{800} f_{n+3} + \frac{27}{6272} f_{n+4} \end{array} \right) \quad (23)$$

$$y_{n+4} = y_n + 4hy'_n + 8h^2 y''_n + \frac{32}{2}h^3 y''' + h^4 \left(\begin{array}{l} \frac{4864}{2835} f_n + \frac{65536}{14175} f_{n+1/2} + \frac{1024}{405} f_{n+1} + \frac{4864}{2835} f_{n+2} \\ + \frac{1024}{14175} f_{n+3} + \frac{32}{2835} f_{n+4} \end{array} \right) \quad (24)$$

$$y'_{n+\frac{1}{2}} = y'_n + \frac{1}{2}hy''_n + \frac{1}{8}h^2 y'''_n + h^3 \left(\begin{array}{l} \frac{65683}{5160960} f_n + \frac{3497}{282240} f_{n+\frac{1}{2}} - \frac{959}{184320} f_{n+1} + \frac{3061}{2580480} f_{n+2} - \frac{397}{1290240} f_{n+3} \\ + \frac{1469}{36126720} f_{n+4} \end{array} \right) \quad (25)$$

$$y'_{n+1} = y'_n + hy''_n + \frac{1}{2}h^2 y'''_n + h^3 \left(\frac{1339}{20160} f_n + \frac{278}{2205} f_{n+\frac{1}{2}} - \frac{2}{63} f_{n+1} + \frac{11}{1440} f_{n+2} - \frac{1}{504} f_{n+3} + \frac{37}{141120} f_{n+4} \right) \quad (26)$$

$$y'_{n+2} = y'_n + 2hy''_n + 2h^2 y'''_n + h^3 \left(\frac{191}{630} f_n + \frac{256}{315} f_{n+\frac{1}{2}} + \frac{52}{315} f_{n+1} + \frac{4}{63} f_{n+2} - \frac{4}{315} f_{n+3} + \frac{1}{630} f_{n+4} \right) \quad (27)$$

$$y'_{n+3} = y'_n + 3hy''_n + \frac{9}{2}h^2 y'''_n + h^3 \left(\frac{1647}{2240} f_n + \frac{486}{245} f_{n+\frac{1}{2}} + \frac{81}{70} f_{n+1} + \frac{729}{1120} f_{n+2} - \frac{9}{280} f_{n+3} + \frac{81}{15680} f_{n+4} \right) \quad (28)$$

$$y'_{n+4} = y'_n + 4hy''_n + 8h^2 y'''_n + h^3 \left(\frac{424}{315} f_n + \frac{8192}{2205} f_{n+\frac{1}{2}} + \frac{128}{45} f_{n+1} + \frac{736}{315} f_{n+2} + \frac{128}{315} f_{n+3} + \frac{8}{441} f_{n+4} \right) \quad (29)$$

$$y''_{n+\frac{1}{2}} = y''_n + \frac{1}{2}hy'''_n + h^2 \left(\frac{2599}{40320} f_n + \frac{311}{3528} f_{n+\frac{1}{2}} - \frac{2693}{80640} f_{n+1} + \frac{601}{80640} f_{n+2} - \frac{31}{16128} f_{n+3} + \frac{143}{564480} f_{n+4} \right) \quad (30)$$

$$y''_{n+1} = y''_n + hy'''_n + h^2 \left(\frac{1499}{10080} f_n + \frac{856}{2205} f_{n+\frac{1}{2}} - \frac{25}{504} f_{n+1} + \frac{83}{5040} f_{n+2} - \frac{11}{2520} f_{n+3} + \frac{41}{70560} f_{n+4} \right) \quad (31)$$

$$y''_{n+2} = y''_n + 2hy'''_n + h^2 \left(\frac{211}{630} f_n + \frac{2048}{2205} f_{n+\frac{1}{2}} + \frac{184}{315} f_{n+1} + \frac{11}{63} f_{n+2} - \frac{8}{315} f_{n+3} + \frac{13}{4410} f_{n+4} \right) \quad (32)$$

$$y''_{n+3} = y''_n + 3hy'''_n + h^2 \left(\frac{579}{1120} f_n + \frac{72}{49} f_{n+\frac{1}{2}} + \frac{369}{280} f_{n+1} + \frac{639}{560} f_{n+2} + \frac{3}{56} f_{n+3} + \frac{9}{7840} f_{n+4} \right) \quad (33)$$

$$y''_{n+4} = y''_n + 4hy'''_n + h^2 \left(\frac{232}{315} f_n + \frac{4096}{2205} f_{n+\frac{1}{2}} + \frac{704}{315} f_{n+1} + \frac{656}{315} f_{n+2} + \frac{64}{63} f_{n+3} + \frac{32}{441} f_{n+4} \right) \quad (34)$$

$$y'''_{n+\frac{1}{2}} = y'''_n + h\left(\frac{4081}{23040}f_n + \frac{77}{180}f_{n+\frac{1}{2}} - \frac{121}{960}f_{n+1} + \frac{313}{11520}f_{n+2} - \frac{1}{144}f_{n+3} + \frac{7}{7680}f_{n+4}\right) \quad (35)$$

$$y'''_{n+1} = y'''_n + h\left(\frac{29}{180}f_n + \frac{24}{35}f_{n+\frac{1}{2}} + \frac{53}{360}f_{n+1} + \frac{1}{120}f_{n+2} - \frac{1}{360}f_{n+3} + \frac{1}{2520}f_{n+4}\right) \quad (36)$$

$$y'''_{n+2} = y'''_n + h\left(\frac{19}{90}f_n + \frac{128}{315}f_{n+\frac{1}{2}} + \frac{14}{15}f_{n+1} + \frac{22}{45}f_{n+2} - \frac{2}{45}f_{n+3} + \frac{1}{210}f_{n+4}\right) \quad (37)$$

$$y'''_{n+3} = y'''_n + h\left(\frac{3}{20}f_n + \frac{24}{35}f_{n+\frac{1}{2}} + \frac{21}{40}f_{n+1} + \frac{51}{40}f_{n+2} + \frac{3}{8}f_{n+3} - \frac{3}{280}f_{n+4}\right) \quad (38)$$

$$y'''_{n+4} = y'''_n + h\left(\frac{14}{45}f_n + \frac{64}{45}f_{n+1} + \frac{8}{15}f_{n+2} + \frac{64}{45}f_{n+3} + \frac{14}{45}f_{n+4}\right) \quad (39)$$

4 Properties of the Methods

Theorem 1

The equation $\pi(r,\overline{h}) = \rho(r) - \bar{h}\sigma(r)$ (40)

where $\rho(r)$ and $\sigma(r)$ are the first and second characteristics polynomials of the method respectively. [11] and [14] posited that a linear multistep is consistent if it satisfies the following conditions

1) The order is $p \geq 1$
2) $\sum_{j=0}^{k} \alpha_j = 0$
3) $\rho(r) = \rho'(r) = \rho''(r) = \rho^{(n-1)}(r) = 0$
4) $\rho^n(r) = n!\,\sigma(r)$ and for the principal root $r = 1$ and n=4

Theorem 2

In the application of k-step scheme to solve problems, we must supply a k initial values $y_{0,}y_1,\ldots,y_{k-1}$ in order to find the solution y_k . These may be calculated from Taylor series, Runge-Kutta or some other method and y_0 is known from initial condition.

Defintion: The linear multistep method of the form (13) is said to be convergent if

$$\lim_{h\to 0} y_j = y(x_j), 0 \leq j \leq N$$

provided that the rounding off errors arising from all the initial conditions tend to zero.

Theorem 3

A Linear Multistep Method of the form (4) is said to be convergent if and only if it is consistent and satisfies the root conditions.

4.1 Order of the methods

Expanding (19) using taylor series gives

$$\begin{bmatrix} \sum_{q=0}^{\infty}\frac{(\frac{1}{2}h)^q}{q!}y^q - y_n - \frac{1}{2}hy'_n - \frac{1}{8}h^2y''_n - \frac{1}{48}h^2y'''_n - \frac{2039}{1161216}h^4y_n^{(iv)} - \sum_{q=0}^{\infty}\frac{h^{q+4}}{q!}y^{q+4}\begin{pmatrix}\frac{4847}{3628800}(\frac{1}{2})^q - \frac{197}{331776}(1)^q + \frac{319}{2322432}(2)^q \\ -\frac{2077}{58060800}(3)^q + \frac{11}{2322432}(4)^q\end{pmatrix} \\ \sum_{q=0}^{\infty}\frac{(h)^q}{q!}y^q - y_n - hy'_n - \frac{1}{2}h^2y''_n - \frac{1}{6}h^3y'''_n - \frac{7181}{362880}h^4y_n^{(iv)} - \sum_{q=0}^{\infty}\frac{h^{q+4}}{q!}y^{q+4}\begin{pmatrix}\frac{2938}{99225}(\frac{1}{2})^q - \frac{853}{90720}(1)^q + \frac{319}{181440}(2)^q \\ -\frac{253}{453600}(3)^q + \frac{187}{2540160}(4)^q\end{pmatrix} \\ \sum_{q=0}^{\infty}\frac{(2h)^q}{q!}y^q - y_n - 2hy'_n - 2h^2y''_n - \frac{4}{3}h^3y'''_n - \frac{536}{2835}h^4y_n^{(iv)} - \sum_{q=0}^{\infty}\frac{h^{q+4}}{q!}y^{q+4}\begin{pmatrix}\frac{45056}{99225}(\frac{1}{2})^q + \frac{8}{2835}(1)^q + \frac{74}{2835}(2)^q \\ -\frac{88}{14175}(3)^q + \frac{16}{19845}(4)^q\end{pmatrix} \\ \sum_{q=0}^{\infty}\frac{(3h)^q}{q!}y^q - y_n - 3hy'_n - \frac{9}{2}h^2y''_n - \frac{9}{2}h^3y'''_n - \frac{621}{896}h^4y_n^{(iv)} - \sum_{q=0}^{\infty}\frac{h^{q+4}}{q!}y^{q+4}\begin{pmatrix}\frac{2214}{1225}(\frac{1}{2})^q + \frac{135}{224}(1)^q + \frac{135}{448}(2)^q \\ -\frac{27}{800}(3)^q + \frac{27}{6272}(4)^q\end{pmatrix} \\ \sum_{q=0}^{\infty}\frac{(4h)h^q}{q!}y^q - y_n - 4hy'_n - 8h^2y''_n - \frac{32}{2}h^3y'''_n - \frac{4864}{2835}h^4y_n^{(iv)} - \sum_{q=0}^{\infty}\frac{h^{q+4}}{q!}y^{q+4}\begin{pmatrix}\frac{65536}{14175}(\frac{1}{2})^q + \frac{1024}{405}(1)^q + \frac{4864}{2835}(2)^q \\ +\frac{1024}{14175}(3)^q + \frac{32}{2835}(4)^q\end{pmatrix} \end{bmatrix} = \begin{bmatrix}0\\0\\0\\0\\0\end{bmatrix}$$

Hence the method is of order seven with error constant

$$C_{p+11} = \left[\frac{-398149}{122624409600}, \frac{-1087477}{23471078400}, \frac{-1768}{3274425}, \frac{-37071}{13798400}, \frac{-3296}{467775}\right]^T$$

4.2 Zero stability of the method

$$\left|\left[\lambda A^{(0)} - A^{(i)}\right]\right| = \left\|\left[\lambda\begin{pmatrix}1&0&0&0&0\\0&1&0&0&0\\0&0&1&0&0\\0&0&0&1&0\\0&0&0&0&1\end{pmatrix} - \begin{pmatrix}0&0&0&0&1\\0&0&0&0&1\\0&0&0&0&1\\0&0&0&0&1\\0&0&0&0&1\end{pmatrix}\right]\right\| = 0$$

$\lambda^5 - \lambda^4 = 0$, $\lambda = 0,0,0,0,1$, hence it is zero stable.

4.3 Consistency

$$y_{n+4} - 14y_{n+2} + 56y_{n+1} - 64y_{n+1/2} + 21y_n = \frac{h^4}{2880}[186f_n - 464f_{n+1/2} + 5761f_{n+1} + 4211f_{n+2} + 375f_{n+3} + 11f_{n+4}]$$

The order, p=9 hence it satisfies p⩾1

i. $\sum_{j=0}^{4}\alpha_j = \alpha_0 + \alpha_{\frac{1}{2}} + \alpha_1 + \alpha_2 + \alpha_4 = 21 - 64 + 56 - 14 + 1 = 0$

ii. $\rho(r) = r^4 - 14r^2 - 56r \ + 64r^{\frac{1}{2}} - 21$

$$\rho(1) = (1)^4 - 14(1)^2 - 56(1) \ + 64(1)^{\frac{1}{2}} - 21 = 0$$

$$\rho'(r) = 4r^3 - 28r \ - 56 + 32r^{\frac{-1}{2}}$$

$$\rho'(1) = 4(1)^3 - 28(1) \ - 56 + 32(1)^{\frac{-1}{2}} = 0$$

Since p(1)= $\rho'(1)$, this shows that condition is satisfied

iii. By the main method, the first and second characteristics polynomial of the method are

$$p(r) = r^4 - 14r^2 - 56r^1 + 64r^{\frac{1}{2}} - 21r^0 = r^4 - 14r^2 - 56r \ + 64r^{\frac{1}{2}} - 21$$

$$\sigma(r) = \frac{186r^0}{2880} - \frac{464r^{\frac{1}{2}}}{2880} + \frac{5761r}{2880} + \frac{4211r^2}{2880} + \frac{375r^3}{2880} + \frac{11r^4}{2880} = \frac{186}{2880} - \frac{464(1)^{\frac{1}{2}}}{2880} + \frac{5761(1)}{2880} + \frac{4211(1)^2}{2880} + \frac{375(1)^3}{2880} + \frac{11(1)^4}{2880} = \frac{10080}{2880} = \frac{7}{2}$$

$$\rho'(r) = 4r^3 - 28r \ + 56 - 32r^{\frac{-1}{2}}$$

$$\rho''(r) = \ 12r^2 - 28 + 16r^{\frac{-3}{2}}$$

$$p(r)''' = 24r \ - 24r^{\frac{-5}{2}}$$

$$p(r)^{\mathbf{iv}} = 24 + 60r^{\frac{-7}{2}}$$

$$p(1)^{\mathbf{iv}} = 24 + 60(1)^{\frac{-7}{2}} = 24 + 60 = 84$$

$$4!\sigma(1) = 4!*\frac{7}{2} = 84$$

For principal root r=1, the conditions (1) - (iv) above are satisfied. Hence the Method is Consistent.

4.4 Stability domain of the method

The Fig. 2 shows the region of stability domain of the method.

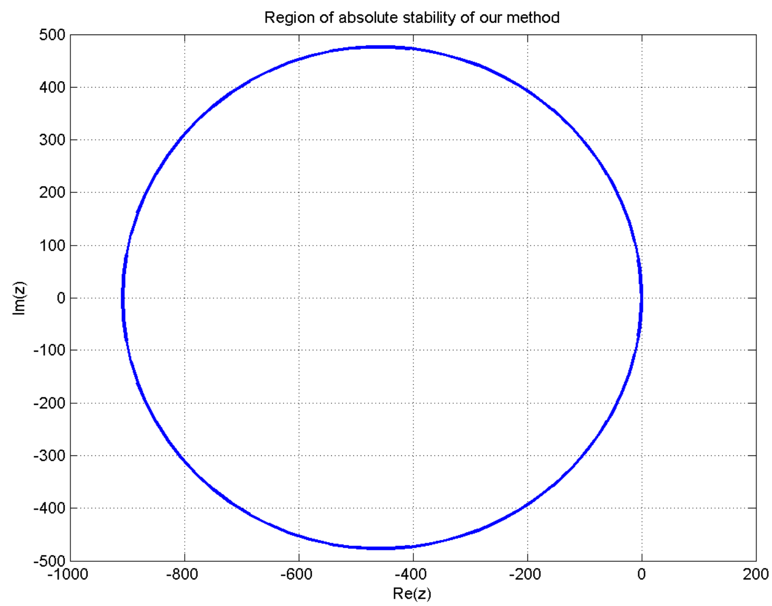

Fig. 2. Shows the stability domain of the method

4.5 Consistence

A Linear Multistep Method of the form (14) is said to be convergent if and only if it is consistent and satisfies the root conditions. Since our method is zero stable and consistent, Hence the method is consistence.

5 Implementation and Numerical Results

5.1 Introduction

This chapter deals with the implementation of the method in solving initial value problems of ordinary differential equation. MATLAB software was used to code the schemes derived and they were tested on some numerical examples ranging from nonlinear to linear initial value problems of fourth order ordinary differential equations. The package comprises of three subroutines; the exact solution, the problem under consideration and the codes.

5.2 Numerical experiments

The methods are tested on some numerical problems to test the accuracy of the proposed methods and our results are compared with the results obtained using existing methods.

The following problems are taken as test problems:

Error = $\left| y(x) - y_n(x) \right|$, where y(x) is the exact solution and $y_n(x)$ is our computed result.

Problem 1: $y^{iv} - 4y'' = 0 \;\; y(0) = 1, y'(0) = 3, y''(0) = 0, y'''(0) = 16 \;\; 0 \le x \le 1 \;\; \text{h} = \frac{0.1}{32}$

Exact solution: $y(x) = 1 - x + e^{2x} - e^{-2x}$

Problem 2: Consider a fourth order differential equation

$$y^{(iv)} = -\sin x + \cos x, y'''(0) = 7, y''(0) = y'(0) = -1, y(0) = 0$$

Exact solution: $y(x) = -\sin x + \cos x + x^3 - 1$

Problem 3: Consider the non-homogeneous linear equation of fourth order below

$$y^{iv} = \frac{-(8 + 25x + 30x^2 + 12x^3 + x^4)}{1 + x^2}$$

$$y(0) = 0 \quad y'(0) = 0 \quad y''(0) = \frac{1}{144 - 100\pi} \quad y'''(0) = -3. \quad \text{h} = \frac{1}{320}$$

Problem 4: Consider the non-linear equation of fourth order below

$$y^{iv} = (y')^2 - yy''' - 4x^2 + e^x(1 - 4x + x^2)$$

$$y(0) = 1 \quad y'(0) = 1 \quad y''(0) = 3 \quad y'''(0) = 1 \quad \text{h} = \frac{0.1}{32}$$

Exact solution: $y(x) = x^2 + e^x$

Problem 5: The propose methods are applied to solve a physical problem from ship dynamics, As stated by [22], when a sinusoidal wave of frequency Ω passes long a ship or offshore structure, the resultant fluid actions vary with time t. In particular case study by [22], the fourth order problems is defined as $y^4 + 3y'' + y(2 + \varepsilon \cos(\Omega t) = 0, \ \ t \succ 0$

Is subjected to the following initial conditions:

$$y(0) = 1, \ y'(0) = 0, \quad y''(0) = 0, \quad y'''(0) = 0, \quad \text{h} = 0.1$$

Where ε=0 for the existence of the theoretical solution, $y(t) = 2\cos t - \cos(t\sqrt{2})$.

Table 1. Showing comparison of the block and pc-method for accuracy of Problem 1

X	Exact result	Computed result	Error in P=7 in block method	Error in P=7 in P-C method
0.103125	1.009375081380367279	1.009375081380367264	1.5000E-17	9.02999935E-10
0.206250	1.018750651046752949	1.018750651046752800	1.4900E-16	1.193999509E-09
0.306250	1.028127197304249133	1.028127197304248000	1.133E-15	2.63997933E-10
0.406250	1.037505208496096172	1.037505208496096000	1.72E-16	2.090004628E-09
0.506250	1.046885173022758589	1.046885173022758387	2.02E-16	1.5736326312E-08
0.603125	1.056267579361003297	1.056267579361001125	2.172E-15	3.2504852554E-08
0.703125	1.065652916082980786	1.065652916082977682	3.104E-15	5.3027433428E-08
0.803125	1.075041671875310031	1.075041671875305141	4.890E-15	7.7515009449E-08
0.903125	1.084434335558167877	1.084434335558156293	1.1584E-14	1.16205765412E-07
1.003125	1.093831396104383644	1.093831396104360521	2.3123E-14	1.60622514555E-07

Table 2. Showing comparison of the block and pc-method for accuracy of Problem 2

X	Exact result	Computed result	Error in P=7 in block method	Error in P=7 in P-C method
0.103125	-0.003129847204687696	-0.00312984720468770183	5.835E-18	9.99029171E-13
0.206250	-0.006269246355772101	-0.00626924635577214781	4.6712E-17	2.009766650E-12
0.306250	-0.009417983687528419	-0.0094179836875288569	4.37479E-16	1.2049037232E-11
0.406250	-0.012575845339462682	-0.0125758453394627160	2.334E-16	3.011813294E-11
0.506250	-0.015742617356611092	-0.0157426173566113317	2.3920E-16	6.303492515E-11
0.603125	-0.018918085689843282	-0.0189180856898435642	2.8020E-16	1.2285909351E-10
0.703125	-0.022102036196162510	-0.0221020361961631824	6.7177E-16	2.2019263855E-10
0.803125	-0.025294254639009744	-0.025294254639010215	4.6706E-16	3.5856212036E-10
0.903125	-0.028494526688567489	-0.0284945266885679628	5.1408E-16	6.6871090638E-10
1.003125	-0.003129847204687696	-0.00312984720468770183	5.835E-18	9.99029171E-13

Table 3. Showing comparison of the block and pc-method for accuracy of Problem 3

X	Exact result	Computed result	Error in P=7 in block method	Error in P=7 in P-C method
0.103125	0.003124984709384509654	0.00312498470906375289 5	8.10409458E-13	3.20756759E-13
0.206250	0.006249877419867115609	0.006249877418478994312	1.462220024E-12	1.388121297E-12
0.306250	0.009374585428699527371	0.009374585422646254469	5.594614251E-12	6.053272902E-12
0.406250	0.01249901526120470559	0.01249901523448523108	9.94494882E-12	2.671947451E-11
0.506250	0.01562307266625029347	0.01562307258251590126	9.83374792E-12	8.373439221E-11
0.603125	0.01874666261169938875	0.01874666241273487053	6.714740119E-11	1.9896451822E-10
0.703125	0.02186968927983855276	0.02186968885452489943	1.9858384226E-10	4.2531365333E-10
0.803125	0.02499205606278295300	0.02499205523539009843	4.5222548996E-10	8.2739285457E-10
0.903125	0.02811366555785853456	0.02811366406222176380	8.0390212338E-10	1.49563677076E-09

Table 4. Showing comparison of the block and pc-method for accuracy of Problem 5

X	Exact result	Computed Result	Error in P=7 in block method	Error in P=7 in P-C method
0.103125	1.003139653527739149	1.003139653526590265	1.148884E-12	9.02145880E-10
0.206250	1.006308634503762010	1.006308634484910541	1.8851469E-11	1.216821428E-09
0.306250	1.009506973589071086	1.009506973491267734	9.780335210E-11	1.21681228E-09
0.406250	1.012734701540634377	1.012734701224070173	3.16564204E-10	1.713796095E-09
0.506250	1.015991849211685747	1.015991848420781680	7.90904067E-10	1.481970916E-08
0.603125	1.019278447552026225	1.019278445875931873	1.676094352E-09	3.058338503E-08
0.703125	1.022594527608326245	1.022594524439161283	3.169164962E-09	4.941858156E-08
0.803125	1.025940120524428841	1.025940115012282396	5.512146445E-09	7.128679089E-08
0.903125	1.029315257541653783	1.029315248546379403	8.995274380E-09	1.058773080E-07
1.003125	1.032719969999102671	1.032719956040014201	1.395908847E-08	1.445520074E-07

Table 5. Accuracy Comparison of K = 4 in Block method of order 7 with Predictor-corrector method of the same order using Problem 5 with h=1/320
(Application problem arising from ship dynamics)

X	Y-exact	Y-approximate	Error in block method, P=7	Error in PC-method, P=7
0.003125	0.999999999992052690	0.999999999992721270	6.685763E-13	5.685763E-10
0.006250	0.999999999872843830	0.999999999858258940	1.458489E-11	1.767654E-10
0.009375	0.999999999356275480	0.999999999247978670	1.082968E-10	5.909878E-09
0.001250	0.999999997965526630	0.999999997573746360	3.917803E-10	5.767654E-09
0.015625	0.999999995033067470	0.999999994007922390	1.025145E-09	1.100202E-08
0.018750	0.999999989700679490	0.999999987483360500	2.217319E-09	6.898767E-08
0.021875	0.999999980919479500	0.999999976693411560	4.226068E-09	4.636354E-08
0.025000	0.999999967449951230	0.999999960091932040	7.358019E-09	5.787654E-07
0.028125	0.999999947861981100	0.999999935893296500	1.196868E-08	2.245763E-07
0.031250	0.999999920534901050	0.999999902072415740	1.846249E-08	2.846249E-07

6 Discussion of Results and Conclusion

Tables 1 – 5 shows comparison of the method implemented in both Block method and Pc- Method. Table 5 is the application problem, a problem from ship dynamic, all the symbols have their usual meaning. It is very clear that the Block method is more accurate in term of accuracy than the Predictor-corrector method. It is also important to note that, this research shows that block method is an alternative way of solving differential equation directly without using Predictor- Corrector method or conventional method. Of great interest are the basics properties of the method, which shows that it is consistence, convergence and zero stable. Hence this methods are recommended for general purpose in all areas of sciences and engineering.

Competing Interests

Authors have declared that no competing interests exist.

References

[1] Adegboye ZA, Yahaya YA. Formulation of a symmetric implicit super Runge-Kutta-Nystrom method for the integration of third order ODEs. Journal of the Nigeria Mathematics Association of Mathematical Physics. 2012;22:425-430.

[2] Adesanya AO, Momoh AA, Adamu M, Tahir A. Five steps block method for the solution of fourth order ordinary differential equations. International Journal of Engineering Research and Applications (IJERA). 2012;2(5):991-998.
ISSN: 2248-9622
Available:www.ijera.com

[3] Badmus AM, Yahaya YA. A class of collocation methods for general second order ordinary differential equations. African Journal of Mathematics and Computer Science Research. 2010;2(4): 069-072.

[4] Boyce Williams, Diprima Richard. Elementary differential equation and boundary value problems 7th Edition. John Wiley & Sons. Inc. New York; 2001.

[5] Butcher JC, Jackiewicz Z. Construction of general linear method with runge-kutta stability properties. Numerical Algorithms. 2004;36:53-72.

[6] Bun RA, Varsolyer YD. A numerical method for solving differential equation of any orders. Journal of Computer and Mathematics Physics. 1992;32(3):317-330.

[7] Dalquist G, Bjock A. Numerical methods, prentice and hall. Englewoods Cliff, New Jersey; 1974.

[8] Fatunla SO. Block methods for second order initial value problems. International Journals of Computer Mathematics. 1991;55-63.

[9] Henrici P. Discrete variable method in ordinary differential equations. John Wiley and Sons, New York; 1962.

[10] Jator SN. A sixth order linear multistep method for the direct solution of $y'' = (x, y, y')$, International Journal of Pure and Applied Mathematics. 2007a;40:457-472.

[11] Lambert JD. Computational methods in ordinary differential equations. John Wiley & Sons; 1973.

[12] Areo EA, Adeniyi RB. Sixth-order hybrid block method for the numerical solution of first order initial value problems. Journal of Mathematical Theory and Modeling. 2013;3.

[13] Jator SN, Li J. A self-starting linear multistep method for a direct solution of general second order initial value problem. Intern. Journal computer and Mathematics. 2009;86:827-836.

[14] Omole EO. Some Implicit Hybrid Numerov type block methods for direct solution of fourth order ordinary differential equations. M.Tech Thesis, Federal University of Technology, Akure, Ondo State, Nigeria; 2016.

[15] Kuboye JO, Omar Z. New zero-stable block method for direct solution of fourth order ordinary differential equations. Indian Journal of Science and Technology. 2015;8:12.

[16] Jator SN. On a class of hybrid methods for $y'' = (x, y, y')$. International Journal of Pure and Applied Mathematics. 2007b;59:381-395.

[17] Olabode BT. Some linear multistep methods for special and general third order initial value problems. Ph.D Thesis, The Federal University of Technology, Akure; 2007. (Unpublished)

[18] Sirisena UW, Onumanyi P. A modified continuous Numerov method for second order ODEs. Nigerian Journal of Mathematics and Application. 1994;7:123–129.

[19] Oluwaseun A, Zurni O. A new algorithm for developing block methods for solving fourth order ordinary differential equations. Global Journal of Pure and Applied Mathematics. 2016;12:1465-1471. ISSN: 0973-1768

[20] Omar Z, Suleiman M. Parallel R-point implicit block method for solving higher order ordinary differential equation directly. Journal of ICT. 2003;3(1):53–66.

[21] Simos TE. Dissipative trigonometrically-fitted methods for second order IVPs with oscillating solutions. International Journal of Mod. Phys. 2002;13(10):1333-1345.

[22] Wu XJ, Wang Y, Price WG. Multiple resonances, responses, and parametric instabilities in offshore structures. Journal of Ship Research. 1988;32(4):285-296.

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