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A Logistic Differential Equation Model Rendition of Customers' Consumption of Electrical Energy

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Authors' contributions

All authors collaborated to bring this work into reality. Author EKA designed the study, performed the mathematical analysis and wrote the first draft of the manuscript. Author JAA managed the analyses and editing of the study. Author SLM worked on the literature. All authors read and approved the final manuscript.

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Abstract

This paper presents a logistic differential equation model of customers' consumption of electrical energy in Ghana. The objective is to model the industrial and commercial consumption of electrical energy of customers of the Electricity Company of Ghana in the Sekondi-Takoradi Metropolis of the Western Region. The paper applies a model based on the Logistic Differential Equation. The consumption data of customers were obtained through an Automatic Meter Reading System which enables a remote reading from customer's energy meter.
The rate of change of energy consumption has been expressed in the form of the Logistic Differential Equation. Analytical solution has been obtained and constants estimated by fitting a historical energy consumption data to a linear regression equation. The carrying capacity of the Logistic equation referred to as the Optimal Asymptote in this paper has been obtained using the Fibonacci Search Technique. All computations were done based on algorithms which were implemented using the C# Programming Language.

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A forecast of a customer's electricity consumption has been done. The forecast consumption was compared with the actual historical consumption in order to ascertain the level of disparity of the forecast from the actual. The Mean Absolute Percentage Error, which measures the forecasting accuracy or predictive power of the model, has been estimated to be ±6.77%. Practically, the model has predicted correctly to the precision of a maximum of 10% above and a minimum of 2% below the historical energy consumption data.

Keywords: Automatic meter reading system; Fibonacci search technique; industrial energy consumption; linear regression; logistic differential equation; mean absolute percentage error; optimal asymptote.

1 Introduction

The Electricity Company of Ghana Limited is a company, wholly owned by government of Ghana, with the mandate to distribute electricity within the southern half of the country. The company has two main classes of energy consumers; these are domestic consumers and commercial/industrial consumers. The second group is further classified based on the level of energy consumption. These consumption classes comprise Bulk Supply Point Consumption (BSP), Boundary Consumption (Boundary), Special Load Tariff Consumption (SLT) and Commercial Tariff – Non Special Load Tariff Consumption (CT-Non SLT).

These classes of consumers constitute a very critical group to the company. As a result of their level of energy consumption and the important role that they play in the economic development of the country, their relevance cannot be overemphasized. The commercial/industrial energy consumption data constitutes the basis for this paper.

In the past, ECG relied on personnel who were deployed to customer premises to read meters, after which the energy consumption data were sent to the respective regional offices for customer bill to be generated. This approach constituted a phase in metering technology development which had a lot of challenges such as the accuracy and reliability of energy consumption data. In an attempt to inject a high level of efficiency and the application of modern technology into the collection of energy consumption data, the Electricity Company of Ghana Limited embarked on a project which involved the deployment of smart meters for industrial/commercial consumers. This deployment has been limited to the southern half of the country and accompanied by the implementation of an Automatic Meter Reading (AMR) System. The smart meters are programmable energy meters that can be programmed to function in a specified manner. It has the capability of reading and storing energy consumption data, voltages, currents and power factors. The meter has a communication modem attached to it, which enables communication between the meter and a communication point at the company's head office. With the aid of the Automatic Meter Reading System and the communication infrastructure that has been put in place, it is possible to remotely read energy consumption data from the meters. The meter readings are stored in a central database at the company's head office. The implementation of the Automatic Meter Reading System (AMR), and the deployment of smart meters for commercial/industrial consumers across the southern half of the country has now given the company the leverage to remotely read enormous amount of energy consumption data into a central database at the company's head office. These readings are done at a relatively low cost. However, all that the company does with the data at the moment is to extract the readings on a monthly basis for the generation of customers' bills. The company does not take advantage of other meaningful pieces of information that can be generated from the huge data available. The major concern however, relates to underutilization of data, probably ascribable to lack of well-defined, accurate and reliable concepts and techniques that can be used to analyze the data that is available.

It is therefore prudent and relevant that a well-defined mathematical model be developed for the purpose of analyzing the data. The model should be tailored at analyzing the data in order to generate results that are targeted at meeting the forecasting needs of the company's management team. Forecast in the sense that it should be possible to analyze energy consumption trend of a customer within a specified time range. Based on that analysis, it should be possible to forecast future energy consumption trend of the customer. Hence, the central objective of this paper revolves around the development of a mathematical model based on the Logistic Differential Equation model in [1] (model A) or [2]. Specifically, the paper intends to achieve the above objective by computing the constants and parameters of the model. This involves analyses of historical energy consumption data using Linear Regression Analysis and Fibonacci Search Technique. Future energy consumption trend will therefore be estimated using the developed model.

As a result of the unavailability of a model that can be used to forecast with high level of accuracy, the future energy demand of consumers, the significance of this paper cannot be overemphasized. By virtue of the paper, it will be relatively easier to plan ahead of time, the nature of distribution networks that must be implemented. The model will equally help to determine the capacity of transformers and switch gears that must be put in place in order to cater for future energy demand at an optimized cost to the company. Also, the model will eventually help to minimize the commercial and technical losses that the company is currently facing, which at the moment stands at about 23%. Furthermore, major management challenges such as the inability to forecast future energy consumption and the difficulty in identifying consumption trend could easily be surmounted supposing the developed model is applied in the analysis of available data. Forecasting energy consumption forms a major factor that informs the nature of distribution networks that must be implemented and the capacity of transformers that must be installed.

It has become eminence to point out to the fact that data that will be used is limited to the consumption data that is read remotely from smart meters across the southern half of the country into the company’s central database. These readings as stated earlier, are done using the Automatic Meter Reading System. The limitation arises out of the fact that the Automatic Meter Reading System has been implemented for only industrial/commercial consumers.

2 Materials and Methods

The model is based on three mathematical concepts. The main concept involves an application of the Logistic growth model. The second concept involves the use of the Fibonacci Search Technique in computing the value of the optimal asymptote (carrying capacity) of the logistic equation. This can be done using the historical energy consumption data obtained. The third concept comprises an application of linear regression to compute the constants in the logistic equation. These constants have been computed by treating the energy consumption data as a time series data. The final solution obtained after the computation of the carrying capacity and constants has been used to extrapolate future energy consumption.

The assumptions which form the basis of the model as in [2] are:

- No new factors influencing the rate of growth different from those during historical period shall come into play in the forecast region. One main factor being the tariff class of consumers.
- The forecast portion of the curve cannot be used to assess the adequacy of the curve to describe the historical growth.
- The value of the optimal asymptote (carrying capacity) will be revised whenever additional data is available.
- The historical data used is assumed to have some form of logistic growth.

2.1 Mathematical formulation

The fundamental equation in the formulation of the model is the logistic growth model which can be represented in its differential equation form as

$$\frac{dE}{dt} = k_1 E\left(1 - \frac{E}{E_{max}}\right) \tag{1}$$

We define the variables, constants and parameters as follows:

- E is the energy consumption which is dependent on time, t months.
- E_{max} represents the expected maximum monthly energy consumption of a consumer whose consumption is being forecasted. In the logistic equation, this parameter is used as the carrying capacity and referred to as optimal asymptote in this paper. This can be computed by applying the Fibonacci Search Technique to the historical energy consumption data of a consumer.
- k_1 and k_2 are constants. These can be computed by regression analysis. In computing these constants, the logistic growth curve must be transformed into a linear equation and fitted to the historical energy consumption data.

The solution to the differential equation above can be found using the analytical approach as demonstrated below. Re-arranging (1) gives the equation

$$\frac{E_{max}dE}{E(E_{max}-E)} = k_1 dt \tag{2}$$

Expressing the coefficient of dE in (2) as partial fraction, we obtain

$$\frac{E_{max}}{E(E_{max}-E)} = \frac{A}{E} + \frac{B}{(E_{max}-E)} \tag{3}$$

Here, A and B are Constants. From (3) we obtain $A = 1$ and $B = 1$.

Thus, (3) reduces to

$$\frac{E_{max}}{E(E_{max}-E)} = \frac{1}{E} + \frac{1}{(E_{max}-E)} \tag{4}$$

Hence, (2) becomes

$$\frac{E_{max}dE}{E(E_{max}-E)} = \left(\frac{1}{E} + \frac{1}{(E_{max}-E)}\right) dE = k_1 dt \tag{5}$$

Consequently,

$$\int \frac{dE}{E} + \int \frac{dE}{(E_{max}-E)} = \int k_1 dt$$

$$ln|E| - ln|E_{max}-E| = k_1 t + k_2$$

$$ln\left|\frac{E}{(E_{max}-E)}\right| = k_1 t + k_2 \tag{6}$$

Further simplification gives

$$E = \frac{E_{max}}{1 + e^{-(k_1 t + k_2)}} \tag{7}$$

We will use (7) to forecast energy consumption for a given time *t*. This equation will be used after the computation of the parameters and constants E_{max} , k_1 and k_2 . Once these parameters and constants are computed the forecast for time $t_1,\ t_2,\ t_3,\ t_4\ ...\ t_n$ can be done.

The computation of (7) is based on an algorithm written and implemented in C Sharp (C#) programming language.

2.2 Equilibrium solution of model

Under this section, we perform qualitative analysis of (1) in order to establish the expected consumption growth behaviour. If we rewrite (1), we have

$$\frac{dE_c}{dt} = k_c E_c \left(1 - \frac{E_c}{E_{max}^c}\right)$$

The equilibrium solution is obtained by solving the equation

$$k_c E_c \left(1 - \frac{E_c}{E_{max}^c}\right) = 0$$

Resulting in the following two equations

$$E_c = 0 \quad \text{and} \quad \left(1 - \frac{E_c}{E_{max}^c}\right) = 0$$

Consequently, we obtain two solutions which are given as

$$E_c = 0 \quad \text{and} \quad E_c = E_{max}^c$$

Ideally, a consumer whose consumption is being forecasted is expected to range between zero consumption and the maximum projected consumption which has been given as E_{max}^c.

2.3 Computation of model parameters and constants

The constants are first computed using linear regression analysis followed by the computation of the optimal asymptote (E_{max}) using the Fibonacci Search Technique. The constants need to be computed first because the computation of the optimal asymptote will require the use of the constants. The computation can be done in four steps as illustrated below:

Step 1: Retrieval of data for analysis

The set of historical energy consumption data which will form the basis for the computation can be obtained and represented as

$$H = \{E_t \,|E_t \in R, t = 1,2,3, \dots, n\} \tag{8}$$

By definition, E_t is the historical monthly energy consumption and t is the time in months.

In order to identify a very important and much needed trend that exists in the historical consumption, smoothening must be done on the data using the Weighted Moving Average [3] of the time series. The weighted moving average algorithm can be used in smoothening out the historical consumption data before the remaining computation can be done. The weighted moving average is given as

$$E_t^{wma} = \frac{tE_t + (t-1)E_{t-1} + \cdots + 2E_2 + E_1}{t + (t-1) + \cdots + 2 + 1} \tag{9}$$

Here, $t = 1, 2, 3, \dots, n$ and E_t^{wma} is the weighted moving average at t.

The implementation of the algorithm that extracts data from the database has been accomplished in C# programming language. The algorithm for the Weighted Moving Average was incorporated in that of the data retrieval. We have also implemented an algorithm for the data range generation using same programing language. For example, if the starting date is 23-03-2013 and the ending date is 20-07-2013, this algorithm will generate the following dates: {31-03-2013, 30-04-2013, 31-05-2013, 30-06-2013, and 31-07-2013}. These dates are necessary in order to be able to extract the energy readings for specific months, since each month's reading in the database is stored against the last date of that month. Still on the data, we have developed two algorithms one for the determination of array size while the other, for the generation of the last date of a given month. The former produces an integer value which is a count of the number of months in the date range. However, the later computes the date of the last day of a given month.

Step 2: Computation of autocorrelation coefficient

In applying the projection technique to a time series analysis, it is assumed that the data values given in step 1 are related to each other at one or more time periods apart. The extent of the relationship can be measured by taking two data sequences from the time series, one lagging the other by one or more time periods and calculating the autocorrelation coefficient between the two sequences. The value of the autocorrelation coefficient ranges from -1 to +1 [4]. For the purpose of this paper, the closer it is to +1, the better the relationship between the data sequence. For the time series with *n* data points given in step 1, the autocorrelation coefficient (r_k) for two data points with *k* periods apart is represented as

$$r_k = \frac{\sum_{t=1}^{n-k}\left(E_t - \overline{E}\right)\left(E_{t+k} - \overline{E}\right)}{\sum_{t=1}^{n}\left(E_t - \overline{E}\right)^2} \tag{10}$$

$$-1 \le r_k \le 1\,, r_k \in R$$
$$1 \le k < n\,, k \in Z$$

We define E_t as the energy consumption for the t^{th} month; and $\overline{E}$, the average value of the historical energy consumption of the time series.

The average value of the time series can be specified by

$$\overline{E} = \frac{1}{n}\sum_{t=1}^{n} E_t \tag{11}$$

The autocorrelation correlation coefficient has been computed by an algorithm whose implementation has been done in the usual C# programming language. The historical consumption data that is extracted from the database has been passed to this algorithm to compute the autocorrelation coefficient. This precedes another algorithm, also implemented in C#, which computes the average consumption of a given historical energy consumption of the time series.

Step 3: Computation of constants

The linear form of the Logistic equation (*6*) as stated below, can be applied to the historical energy consumption data given is step 1 to compute k_1 and k_2 via regression analysis [5].

$$ln\left|\frac{E}{(E_{max} - E)}\right| = k_1 t + k_2$$

Accordingly, the constants k_1 and k_2 can be computed as follows:

$$k_1 = \frac{n \sum tE - (\sum t)(\sum E)}{n(\sum t^2) - (\sum t)^2} \tag{12}$$

$$k_2 = \frac{\sum E - k_1 \sum t}{n} \tag{13}$$

Practically, k_1 refers to the rate of change in energy induced by a change in time while k_2 defines the amount of energy consumed at the initial stage when time is zero. The implementation of the algorithm for computing the constants k_1 and k_2 has been written in C# programming language.

Step 4: Computation of optimal asymptote

The computation of the optimal asymptote can be executed using the Fibonacci Search Technique which is an optimization technique. The objective function to be minimized is the Sum of Squared Residual (SSR). The process has been illustrated as follows:

SSR and MAPE computation

When the model is fitted to historical energy consumption data set as in the case of the linear form of the logistic equation, the model is assessed on the basis of how well it fits the historical energy consumption data. This is referred to as the *goodness of fit* or best fit as given in [6]. How well it could be used to estimate future consumption values is referred to as the "forecasting accuracy" [7,8,9,10]. The "goodness of fit" can be computed as the sum of squared residuals (SSR) specified as follows:

$$SSR = \sum_{t=1}^{n} \left(\overline{E_t} - E_t\right)^2 \tag{14}$$

The forecasting accuracy can be computed as the mean absolute percentage error (MAPE) [8,9,10] which is given as

$$MAPE = \frac{1}{n} \sum_{t=1}^{n} \left| \left(\frac{E_t - \overline{E_t}}{E_t} \times 100 \right) \right| \tag{15}$$

We define the variables as given below:

- E_t is the actual consumption value
- $\overline{E_t}$ is the corresponding predicted values, and
- n is the number of data points used.

We obtained our computational values for (14) and (15) by implementing their respective algorithms in C# programming language.

2.4 Fibonacci number generation

In a single variable case [8] where the assumption of unimodality holds, an application of the Fibonacci Search Technique [11] is warranted. While locating the optimal asymptote (E_{max}), the sum of the squared residuals (SSR) is minimized between the fitted curve and the actual data. The Fibonacci Search Technique involves the use of Fibonacci numbers and can be generated by the expression

$$F_k = F_{k-1} + F_{k-2} \quad for\ k > 1 \ \ with\ F_0 = 1\ and\ F_1 = 1 \tag{16}$$

The first k Fibonacci numbers are generated, where the number of terms k determines the number of iterations that will be performed in the computation of the optimal asymptote. The value of k can be computed from the relation

$$\frac{1}{F_k} < \frac{r}{100} \tag{17}$$

Where r % is the interval of uncertainty and F_k is the k^{th} Fibonacci term.

To generate the Fibonacci number, we implemented an algorithm in the C# programming language. Using the Fibonacci Search Technique in computing the optimal asymptote (E_{max}), we proceed as follows:

1. A range of values of the energy consumption data E is established, within which the optimal asymptote (E_{max}) could be located. The lower limit, denoted as E_L, represents the maximum monthly energy consumption value recorded. It has been obtained from the known historical energy consumption data. The upper limit E_U can be computed as 100 times the value of E_L. If no solution of E is found within the selected range, then it can be concluded that the historical energy consumption data has little influence on the optimal asymptote.
2. Two equidistant points E^I and E^{II} from each end of the interval can be established as shown in Fig. 1.

$$E^I = E_L + d_1 \tag{18}$$

$$E^{II} = E_U - d_1 \tag{19}$$

The distance d_1 is defined as

$$d_1 = \frac{F_{k-2}}{F_k} \times L \tag{20}$$

$$L = E_U - E_L \tag{21}$$

By definition, F_{k-2} is the $(k-2)^{th}$ Fibonacci number, F_k is the k^{th} Fibonacci number and $(k-1)$ is the number of calculations to be performed. Two calculations of SSR are made at the point E^I and E^{II}, namely SSR_1 and SSR_2 respectively, where

$$SSR_1 = \sum_{t=1}^{n} \left(\overline{E_t} - E_t\right)^2 \tag{22}$$

$$\overline{E_t} = \frac{E^I}{1 + e^{-(k_1 t + k_2)}} \tag{23}$$

$$SSR_2 = \sum_{t=1}^{n} (\overline{E_t} - E_t)^2 \tag{24}$$

$$\overline{E_t} = \frac{E^{II}}{1 + \exp - (k_1 t + k_2)} \tag{25}$$

3. A new interval $E_L^{'}$ and $E_U^{'}$ is established based on the relative magnitudes of SSR_1 and SSR_2 which have been calculated in the previous step. If $SSR_1 < SSR_2$, then the region $E^{II}E_U$ is discarded and a new upper limit is chosen as $E_U^{'} = E^{II}$ and the lower limit is maintained as $E_L^{'} = E_L$. Alternatively,

if $SSR_1 > SSR_2$, then the lower region $E_L E^I$ is discarded such that $E_U^{'} = E^U$ and $E_L^{'} = E^I$. This narrows the interval within which the optimal asymptote would lie based on the minimum *SSR*.

4. The new value of the narrowed interval $L^{'}$ and a new value for the distance d_2 is then calculated as follows:

$$L^{'} = E_U^{'} - E_L^{'} \tag{26}$$

$$d_2 = \frac{F_{k-3}}{F_{k-1}} \times L^{'} \tag{27}$$

The new value of E^I and E^{II} and their corresponding SSR_1 and SSR_2 are computed.

5. Steps 3 and 4 are repeated, replacing old values of d and L with new values each time until all the $(k-1)$ iterations are performed. At the final iteration, the points E^I and E^{II} will be very close together as the interval L is narrows significantly by the Fibonacci Search Technique at each step. At this point the optimal asymptote E_{max} is chosen as the value of *E* with the lowest *SSR*.

Upon the completion of this stage, (7) can now be used to compute the forecast consumption. The Weighted Moving Average algorithm is then further used in smoothening out the forecast consumption data in order to identify the required future consumption trend.

The implementation of the Fibonacci Search Algorithm for computing the value of the optimal asymptote was performed using the C# programming language.

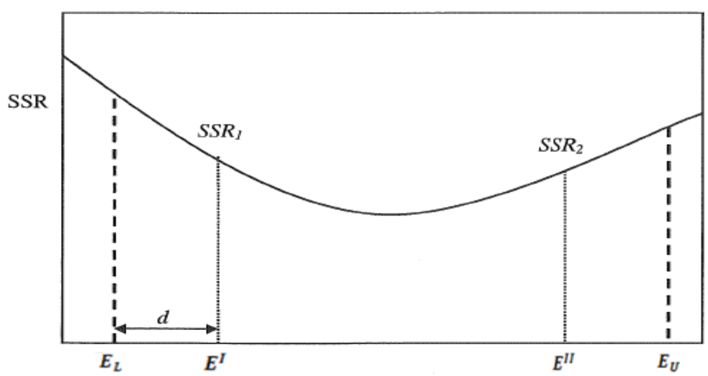

Fig. 1. Minimizing SSR using the Fibonacci search technique

3 Results and Discussion

After the formulation of the model in the previous section, it is necessary to put the model into practice in order to get a clear understanding of how the model works. This chapter focuses on using the model to analyze some sample data which have been extracted from the Automatic Meter Reading database. Computations at the various stages of the model are performed and analysis done to show the predictive potency of the model.

3.1 Analysis of a customer energy consumption

The consumption pattern of a single customer has been analyzed in this section and used to forecast future consumption for thirty-six months. The thirty-six months include the period of the historical consumption data to allow for a comparison of the historical consumption and the forecast consumption. The historical energy consumption data used for the analysis has been available in the Automatic Meter Reading central database where it was extracted for the analysis. The model can however be extended to the total energy consumption of a group of consumers.

We present a historical energy consumption data of a selected consumer (See Table 1). The data ranges from January 2012 to February 2013, a total of 14 months.

Table 1. Historical energy consumption data for a customer

Months	Time (*t*)	Consumption (KWH)
JAN 2012	1	1010.000
FEB 2012	2	1493.060
MAR 2012	3	5000.029
APR 2012	4	6002.968
MAY 2012	5	10003.005
JUN 2012	6	12991.445
JUL 2012	7	14009.799
AUG 2012	8	17089.809
SEP 2012	9	17001.024
OCT 2012	10	22008.991
NOV 2012	11	22991.282
DEC 2012	12	23998.697
JAN 2013	13	25740.014
FEB 2013	14	25970.236

The analysis of autocorrelation reveals that autocorrelation coefficients range from -1 to 1. When the autocorrelation coefficients are high, i.e. close to ± 1, it implies that the data points are closely correlated at the specified time period. However, for the purpose of this paper, the preferred option is when the autocorrelation coefficient is close to +1. This means that it is statistically significant to make a forecast to the next time period based on the present data. A summary of the autocorrelation coefficient values r_k for time lags of $k = 1, 2, 3, 4, \dots, 13$ months interval has been given (as Table 2 presents). The maximum periods apart which is 13 months was chosen to be one month less than the total number of months in the historical energy consumption of the time series. The values give a fairly good idea of the level of correlation between the data points in the time series. It can be noticed that the autocorrelation coefficient is relatively high when the data points are one period apart. It keeps reducing until the data points are four periods apart after which it eventually assumes negative values; but reasonably close to the positives. As a result of the relatively good autocorrelation coefficient for the first few time intervals, the model can be applied to the historical consumption data in Table 1 to give a reasonably good forecast. In computing the autocorrelation coefficient, the average consumption from Table 1 is computed using (11) to obtain $\overline{E} = 14665.026$ KWH . The autocorrelation coefficient from one month up to 13 months period apart is computed by applying (10).

3.1.1 Computation of constants

The linear form of the logistic equation has been fitted to the historical energy consumption data in Table 1 from which the constants k_1 and k_2 are computed using (12) and (13). The following sums have been computed: n = 14, $\sum tE$ = 2017949.917, $\sum t$ = 105.000, $\sum t^2$ = 1015.000 and $\sum E$ = 205310.359. Consequently, we obtain $k_1 = 2101.636$ and $k_2 = -1097.245$. After computing the constants k_1 and k_2, the values were rationalized by dividing k_1 by itself and dividing k_2 by k_1 to obtain $k_1 = 1.00$ and $k_2 = -0.522$. The values needed to be rationalized because the original values of the constants have an unwanted effect on the model results due to their large values.

Table 2. Autocorrelation coefficient for historical energy consumption data

Period (k)	**Correlation coefficient (r_k)**
1	0.8082
2	0.5915
3	0.3890
4	0.1696
5	-0.0129
6	-0.1399
7	-0.2696
8	-0.3515
9	-0.4376
10	-0.4306
11	-0.3731
12	-0.2927
13	-0.1505

3.1.2 Computation of optimal asymptote

Using an interval of uncertainty of 0.010 % produces k *=19* which requires the generation of the first 20 terms of the Fibonacci numbers (refer to Table 3). The energy consumption interval within which the optimal asymptote (E_{max}) could be located has been computed. The lower limit E_L is the maximum consumption value from the historical energy consumption data in Table 1. The maximum consumption is 25970.236 KWH, which implies

$$E_L = 25970.236 \text{ KWH}$$

$$E_U = 100E_L$$

$$E_U = 2597023.600 \text{ KWH}$$

The consumption range E_L to E_U has been used to compute the optimal asymptote (E_{max}) using the Fibonacci Search Technique. From the iteration process of the Fibonacci Search Technique, the optimal asymptote has been computed and given as

$$E_{max} = 37162.591 \text{ KWH}$$

Table 3. First 20 terms of the Fibonacci numbers (F_k)

Record no	F_k	**Record no**	F_k
0	1	10	89
1	1	11	144
2	2	12	233
3	3	13	377
4	5	14	610
5	8	15	987
6	13	16	1597
7	21	17	2584
8	34	18	4181
9	55	19	6765

From the logistic equation, the forecast consumption for 36 months starting from the first month of the historical consumption is shown (Table 4). The optimal asymptote of the logistic equation has significant effect on the forecasts made. Once the historical data have been used to get the best fit, represented by the constants of the regression equation, it is the upper limit that determines the accuracy of the forecast. While the Fibonacci Search Technique used to compute the asymptote has been proven to be an effective method, the value of the asymptote will depend on the extent of the data used in determining its value. The more the available historical consumption data, the more accurate the asymptote value that has been obtained.

Table 4. Thirty-six (36) months forecast energy consumption for customer

Time (months)	Consumption (KWH)	Time (months)	Consumption (KWH)	Time (months)	Consumption (KWH)
JAN 2012	1111.000	JAN 2013	25225.214	JAN 2014	31930.191
FEB 2012	1642.366	FEB 2013	27201.706	FEB 2014	32131.942
MAR 2012	5500.032	MAR 2013	28433.176	MAR 2014	32318.748
APR 2012	6603.265	APR 2013	28979.588	APR 2014	32492.211
MAY 2012	11003.306	MAY 2013	29461.713	MAY 2014	32653.710
JUN 2012	14290.590	JUN 2013	29890.269	JUN 2014	32804.443
JUL 2012	15410.779	JUL 2013	30273.713	JUL 2014	32945.452
AUG 2012	16748.013	AUG 2013	30618.813	AUG 2014	33077.647
SEPT 2012	18701.126	SEPT 2013	30931.046	SEPT 2014	33201.831
OCT 2012	21568.811	OCT 2013	31214.894	OCT 2014	33318.710
NOV 2012	22531.456	NOV 2013	31474.060	NOV 2014	33428.909
DEC 2012	23518.723	DEC 2013	31711.628	DEC 2014	33532.987

We illustrate the trend of the historical and the forecast consumptions plotted on the same set of axis (Fig. 2). From the forecast consumption curve (illustrated with a blue curve), it can be observed that the consumption increases steadily from January 2012 until March 2013 after which the graph slows down in its growth. Between January 2012 and February 2013, the historical consumption has been modeled using the developed model after which the actual forecast has been done from March 2013 onwards.

3.2 Comparison of historical and forecast consumption data

At this stage, we present a comparison of the forecast data produced by the model against the actual historical data (see Table 5). The comparison has been done from January 2012 to February 2013. The first column illustrates the period, the second column shows the forecast consumption generated by the model and the third column denotes the historical consumption data. The fourth column represents the difference in value between the forecast and the historical consumption while the fifth column depicts the percentage difference. The percentage difference has been computed relative to the historical consumption for each period. To give a visual impression, we present a graph (Fig. 3) that spans the period for which historical data have been available. It graphically illustrates a comparison of the historical and forecast data. It could be observed from the table that the forecast value at maximum is 10% more than the historical value and at minimum 2% less than the historical value.

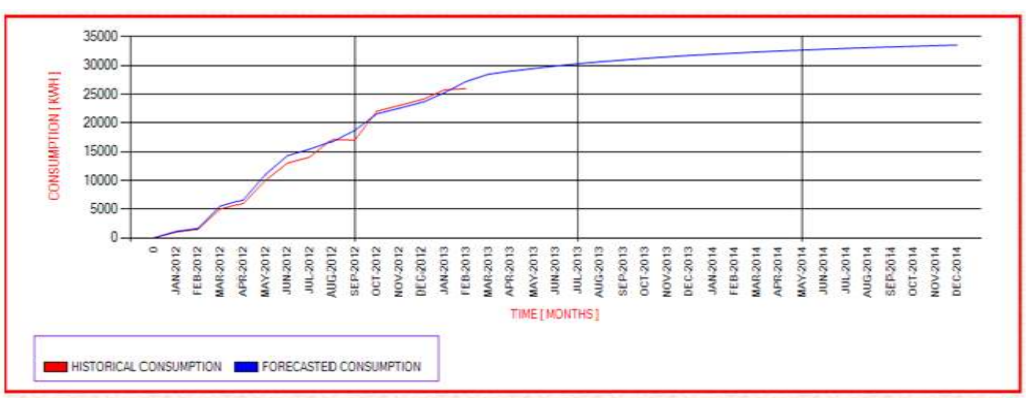

Fig. 2. Graph of time against energy consumption for customer

Table 5. Comparison of historical and forecast energy consumption data

Time (months)	Forecast - F (KWH)	Historical - H (KWH)	(F - H) (KWH)	(F - H)*100 / H (% difference)
JAN 2012	1111.000	1010.000	101.000	10.00
FEB 2012	1642.366	1493.060	149.306	10.00
MAR 2012	5500.032	5000.029	500.003	10.00
APR 2012	6603.265	6002.968	600.297	10.00
MAY 2012	11003.306	10003.005	1000.301	10.00
JUN 2012	14290.590	12991.445	1299.145	10.00
JUL 2012	15410.779	14009.799	1400.980	10.00
AUG 2012	16748.013	17089.809	-341.796	-2.00
SEP 2012	18701.126	17001.024	1700.102	10.00
OCT 2012	21568.811	22008.991	-440.180	-2.00
NOV 2012	22531.456	22991.282	-459.826	-2.00
DEC 2012	23518.723	23998.697	-479.974	-2.00
JAN 2013	25225.214	25740.014	-514.800	-2.00
FEB 2013	27201.706	25970.236	1231.470	4.74

3.3 Qualitative analysis of the forecast data

From (15), the Mean Absolute Percentage Error (MAPE) which is a measure of the forecasting accuracy has been determined. Using the forecast and historical consumption given in Table 5, we obtain MAPE = 6.77. From the value obtained for the MAPE, the forecast consumption on average is expected to have a value of ± 6.77 % of the historical consumption.

The equilibrium solution of the Logistic differential equation (1) has been determined. From the equilibrium solution determined, we have $E = 0$ and $E = E_{max} = 37162.591$.

Based on the equilibrium solution obtained, the forecast consumption is expected to fall in the range of $0 - 37162.591$ KWH.

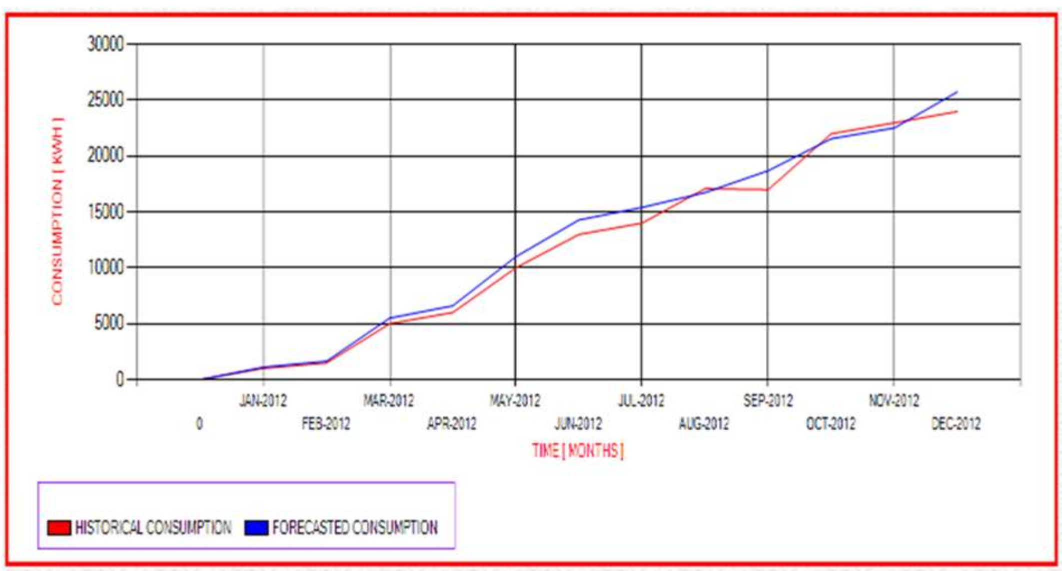

Fig. 3. Comparison graph for historical and forecast energy consumption data

4 Conclusion

The paper has completed the process of modelling commercial and industrial consumption of electrical energy in Ghana. A mathematical model based on the logistic differential equation was used as the basis for the development of the model. The rate of change of energy consumption was expressed in the form of the logistic differential equation, after which the analytical solution for the equation was obtained. The constants in the analytical solution were obtained by analyzing historical energy consumption data using Linear Regression Analysis. The carrying capacity of the logistic equation referred to as the Optimal Asymptote in this paper was obtained using the Fibonacci Search Technique. The Fibonacci Search Technique was equally applied to the historical energy consumption data that was used to compute the constants.

After the computation of the constants and parameter, electricity consumption forecast was done for the customer under consideration. The forecast consumption was compared with the actual historical consumption in order to ascertain the level of disparity of the forecast from the actual. The model performed well in predicting the forecast values as it predicted correctly to the precision of a maximum of 10% above and a minimum of 2% below the historical energy consumption data.

Further we performed qualitative analysis by computing the Mean Absolute Percentage Error which is a measure of the forecasting accuracy of the model. The value obtained for the MAPE on the average is expected to be ± 6.77% of the historical consumption.

This paper was limited to the development of a mathematical model for forecasting electricity consumption of consumers whose historical consumption of electrical energy follows the logistic growth curve. The model generated a good forecast for consumers whose historical energy consumption had logistic growth. Therefore, it is recommended that any future research should concentrate on developing a forecasting model for consumers with non-logistic growth.

Competing Interests

Authors have declared that no competing interests exist.

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