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Eta-Einstein S-Manifolds and Spectral Geometry

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Author's contribution

The sole author designed, analyzed and interpreted and prepared the manuscript.

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Abstract

We examine the spectral geometry of η-Einstein S-manifolds and compute the the spectral coefficients for S-space form and obtain the results analogue to Patodi's results for Riemannian manifolds and J. Park results for $\eta-$Einstein Sasakian manifolds. We show that how an $\eta-$Einstein S-manifold and S-space forms are spectrally determined.

Keywords: S-Manifolds; η- Einstein manifolds; spectral geoemtry; laplace operator.

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1 Introduction

The spectral geometry of the Laplace-Beltrami operator has been studied by many authors [1, 2, 3], [4] and has developed rapidly during the last decade. Mainly, topic of Spectral Geometry is developed by Patodi's work [4] on heat-kernel asymptotic for operators of Laplace type on manifolds with boundary. Gilkey [3] computed some spectral invariants concerning the asymptotic expansion of the trace of the heat kernel for an elliptic differential operator acting on the space of sections

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of a vector bundle. Later on Donnely [1] and Sacks [2] extended the Patodi's results to complex manifolds. After this J. Park extended these result to $\eta-$Einstein contact manifolds [5].

Generalizing the notion of η-Einstein contact manifolds, M. Kobayashi and S. Tsuchiya defined the notion of η-Einstein S-manifolods [6], [7]. For more detail on $\eta-$Einstein S-manifolds see [8].

Let Δ_p be the Laplace Beltrami operator acting on the space of smooth p-forms, p=0, 1, 2 over a compact $m-$dimentional Riemannian manifold M. Patodi [4] set up the following result for the space forms.

Theorem 1.1. *[4] Let (M_i, g_i) be the compact Riemanian manifolds without boundary. Assume that $Spec(\Delta_p, M_1) = Spec(\Delta_p, M_2)$, for $p = 0, 1, 2$. Then*

(1) The manifold M_1 has constant scalar curvature c if and only if the manifold M_2 has constant scalar curvature c.
(2) The manifold M_1 is Einstein if and only if the manifold M_2 is Einstein.
(3) The manifold M_1 has constant sectional curvature c if and only if the manifold M_2. has constant sectional curvature c.

We extend these results to $\eta-$Einstein S-manifolds which are the generalization of both complex and contact structure. For $s = 0$, our results are for manifolds with complex structure, for $s = 1$, we get results for $\eta-$Einstein Sasakian manifolds [5].

In this paper, after introduction, the second section is related to S-manifolds. Third section is related to main results related to curvature tensor and heat trace asymptotics on η-Einstein S-manifolds.

2 S-manifolds

As a generalization of both almost complex (in even dimension) and almost contact (in odd dimension) structures, Yano introduced in [9] the notion of f-structure on a smooth manifold of dimension $2n+s$, i.e. a tensor field of type (1,1) and rank $2n$ satisfying $f^3+f = 0$. The existence of such a structure is equivalent to a reduction of the structural group of the tangent bundle to $U(n) \times O(s)$. Let N be a $(2n + s)$-dimensional manifold with an f-structure of rank $2n$. If there exist s global vector fields $\xi_1, \xi_2, \dots, \xi_s$ on N such that:

$$f\xi_\alpha = 0, \qquad \eta_\alpha \circ f = 0, \qquad f^2 = -I + \sum \xi_\alpha \otimes \eta_\alpha, \tag{2.1}$$

where η_α are the dual 1-forms of ξ_α, we say that the f-structure has complemented frames. For such a manifold there exists a Riemnnian metric g such that

$$g(X, Y) = g(fX, fY) + \sum \eta_\alpha(X)\eta_\alpha(Y)$$

for any vector fields X and Y on N.

An f-structure f is normal, if it has complemented frames and

$$[f, f] + 2\sum \xi_\alpha \otimes d\eta_\alpha = 0,$$

where $[f, f]$ is Nijenhuis torsion of f.

Let F be the fundamental 2-form defined by $F(X, Y) = g(X, fY)$, X, $Y \in T(N)$. A normal f-structure for which the fundamental form F is closed, $\eta_1\wedge, \dots, \wedge\eta_s \wedge (d\eta_\alpha)^n \neq 0$ for any α, and $d\eta_1 = \cdots = d\eta_s = F$ is called to be an S-structure. A smooth manifold endowed with an S-structure will be called an S-manifold. These manifolds were introduced by Blair in [6].

We have to remark that that if we take $s = 1$, S-manifolds are natural generalizations of Sasakian manifolds. In the case $s \geq 2$ some interesting examples are given in [6, 10].

If N is an S-manifold, then the following formulas are true (see [6]):

$$\overline{\nabla}_X \xi_\alpha = -fX, \qquad X \in T(N), \quad \alpha = 1, \dots, s, \tag{2.2}$$

$$(\overline{\nabla}_X f)Y = \sum \{g(fX, fY)\xi_\alpha + \eta_\alpha(Y) f^2 X\}, \quad X, Y \in T(N), \tag{2.3}$$

where $\overline{\nabla}$ is the Riemannian connection of g. Let L be the distribution determined by the projection tensor $-f^2$ and let M be the complementry distribution which is determined by $f^2 + I$ and spanned by $\xi_1, \dots, \xi_s$. It is clear that if $X \in L$ then $\eta_\alpha(X) = 0$ for any α, and if $X \in M$, then $fX = 0$. A plane section π on N is called an invariant f-section if it is determined by a vector $X \in L(x)$, $x \in N$, such that $\{X, fX\}$ is an orthonormal pair spanning the section. The sectional curvature of π is called the f-sectional curvature. If N is an S-manifold of constant f-sectional curvature k, then its curvature tensor has the form

$$\begin{aligned} R(X,Y,Z,W) = \sum_{\alpha,\beta} \{ & g(fX,fW)\eta_\alpha(Y)\eta_\beta(Z) - g(fX,fZ)\eta_\alpha(Y)\eta_\beta(W) + \\ & + g(fY,fZ)\eta_\alpha(X)\eta_\beta(W) - g(fY,fW)\eta_\alpha(X)\eta_\beta(Z)\} + \\ & + \frac{1}{4}(k+3s)\{g(fX,fW)g(fY,fZ) - g(fX,fZ)g(fY,fW)\} + \\ & + \frac{1}{4}(k-s)\{F(X,W)F(Y,Z) - F(X,Z)F(Y,W) - 2F(X,Y)F(Z,W)\}, \end{aligned} \tag{2.4}$$

X, Y, Z, W $\in T(N)$. Such a manifold $N(k)$ will be called an S-space form. The Euclidean space E^{2n+s} and the hyperbolic space H^{2n+s} are examples of S-space forms.

Definition 2.1. [7], [8] S-manifold $(M_m, \eta_\alpha, g, f, \xi_\alpha)$ is said to be η-Einstein if the Ricci tensor ρ of M is of the form

$$\rho = ag + b\sum_{\alpha=1}^{s} \eta_\alpha \otimes \eta_\alpha, \tag{2.5}$$

where a, b are constants on M.

3 Main Results

Let M be a $(2m+s)-$dimensional S-space form then from (2.4), the ricci tensor ρ is given by

$$\rho = \frac{4s + (k+3s)(2m-1) + 3(k-1)}{4} g + \frac{(2m+s-2)(4-k-3s) - 3(k-s)}{4} \eta_\alpha \otimes \eta_\alpha. \tag{3.1}$$

Hence M is an eta-Einstein manifold.

Define the tensor fields $S_{\alpha,\beta}$ and T_c of M respectively by

$$\begin{aligned} S_{a,b}(X,Y) &= \rho(X,Y) - (ag(X,Y) + b\sum_{\alpha=1}^{s} \eta_\alpha(X) \otimes \eta_\alpha(X)), \\ T_c(X,Y,Z,W) &= R(X,Y,Z,W) - \{\sum_{\alpha,\beta}\{g(fX,fW)\eta_\alpha(Y)\eta_\beta(Z) - g(fX,fZ). \\ &. \quad \eta_\alpha(Y)\eta_\beta(W) + g(fY,fZ)\eta_\alpha(X)\eta_\beta(W) - g(fY,fW)\eta_\alpha(X)\eta_\beta(Z)\} \\ &+ \quad \frac{1}{4}(k+3s)\{g(fX,fW)g(fY,fZ) - g(fX,fZ)g(fY,fW)\} \\ &+ \quad \frac{1}{4}(k-s)\{F(X,W)F(Y,Z) - F(X,Z)F(Y,W) - \\ &- \quad 2F(X,Y)F(Z,W)\}\}, \end{aligned} \tag{3.2}$$

for vector fields X, Y, Z, W on M, where a, b are smooth functions on M and k is a constant f-sectional curvature.

Let $\{e_i\}$ be an orthonormal basis of T_pM at any point $p \in M$. Next in this paper, we shall use the following notations:

$$R_{ijkl} = g(R(e_i, e_j)e_k, e_l), \quad \rho_{i,j} = \rho(e_i, e_j), \quad f_{ij} = g(fe_i, e_j), \tag{3.3}$$
$$\nabla_i f_{jk} = g((\nabla_{e_i} f)e_j, e_k), \quad \nabla_i \eta_{\alpha j} = g((\nabla_{e_i} \xi_\alpha, e_j) \tag{3.4}$$

and so on, where indices run over the range $1, 2, \ldots, 2m + s$.

The equations (2.2), (2.3) can be written as:

$$\nabla_i f_{jk} = \sum_\alpha g_{ij}\eta_{\alpha k} - \eta_{\alpha j} g_{ik} \tag{3.5}$$
$$\nabla_i \eta_{\alpha j} = -f_{ij} \tag{3.6}$$

By the definition of the tensor field $S_{\alpha,\beta}$, (3.5), (3.6) and some computations, we have

$$|S_{a,b}|^2 = |\rho|^2 + (2m + s)a^2 + sb^2 - 2a\tau + 2abs - 4bms \tag{3.7}$$

Here we see that M is η-Einstein if and only if $S_{a,b} = 0$ and

$$a = \frac{\tau}{2m} - s, \quad b = s(2m + s) - \frac{s\tau}{2m}. \tag{3.8}$$

Now first we shall prove the following lemma for further computations of $\|T_k\|^2$ of the tensor field T_k on S-manifold M. In case of Sasakian manifolds, it was proved by J. Park [5].

Lemma 3.1. *On S-manifold, we have*

$$R_{ijkl}\{f_{ki}f_{jl} - f_{kj}f_{il} + 2f_{ji}f_{kl}\} = 6\tau - 12ms + 12m(2m + s - 2)$$

Proof. It is clear from the properties of curvature tensor that

$$R_{ijkl}f_{ki}f_{jl} = \frac{1}{2}(R_{ijkl} - R_{kjil})f_{ki}f_{jl} = \frac{1}{2}(R_{ijkl} + R_{jkil})f_{ki}f_{jl} = -\frac{1}{2}R_{kijl}f_{ki}f_{jl}. \tag{3.9}$$

Similarly

$$-R_{ijkl}f_{kj}f_{il} = -\frac{1}{2}R_{jkil}f_{jk}f_{il} \tag{3.10}$$
$$R_{ijkl}f_{ji}f_{kl} = -R_{ijkl}f_{ij}f_{kl} \tag{3.11}$$

from (3.9), (3.10) and (3.11) we have

$$R_{ijkl}\{f_{ki}f_{jl} - f_{kj}f_{il} + 2f_{ji}f_{kl}\} = -3R_{ijkl}f_{ij}f_{kl}. \tag{3.12}$$

Also

$$\nabla_l\nabla_i f_{jk} = \sum_\alpha g_{ij}\nabla_l\eta_{\alpha k} - g_{ik}\nabla_l\eta_{\alpha j} = \sum_\alpha -g_{ij}f_{lk} + g_{ik}f_{lj}, \tag{3.13}$$

$$\nabla_l\nabla_i f_{jk} - \nabla_i\nabla_l f_{jk} = \sum_\alpha -g_{ij}f_{lk} + g_{ik}f_{lj} + g_{lj}f_{ik} + g_{lk}f_{ij}. \tag{3.14}$$

Apply Ricci identity to (3.14) and taking sum by setting $i = k$ in the resulting equality, we get

$$-R_{lij\beta}f_{\beta i} - \rho_{l\beta}f_{j\beta} = (2m + s - 2)f_{lj}, \tag{3.15}$$

and

$$\begin{aligned} -R_{lij\beta}f_{\beta i}f_{lj} - \rho_{l\beta}f_{j\beta}f_{lj} &= (2m+s-2)f_{lj}f_{lj} \\ &= (2m+s-2)(2m). \end{aligned}$$

Hence

$$\begin{aligned} -R_{il\beta}jf_{\beta i}f_{jl} &= -\rho_{l\beta}(g_{l\beta} - \eta_l\eta_\beta) + (2m+s-2)(2m) \\ &= -\tau + 2ms + (2m+s-2)(2m). \end{aligned} \tag{3.16}$$

From (3.9) and (3.16)

$$\frac{1}{2}R_{jkil}f_{jk}f_{il} = -\tau + 2ms + (2m+s-2)(2m). \tag{3.17}$$

From (3.12) and (3.17)

$$R_{ijkl}\{f_{ki}f_{jl} - f_{kj}f_{il} + 2f_{ji}f_{kl}\} = 6\tau - 12ms + 12m(2m+s-2). \quad \square$$

3.1 Heat trace asymptotics

Let M be a compact Riemannian manifold of real dimension m without boundary and let D ba a operator of Laplace type on the space of smooth sections to a smooth vector bundle over M. Let e^{-tD} be the fundamental solution of the heat equation. This operator is of trace class and as $t \downarrow 0$ there is a complete asymptotic expansion with locally computable coefficients in the form:

$$Tr_{L^2}e^{-tD} \sim \sum_{n\geq 0} t^{(n-m)/2}a_n(D). \tag{3.18}$$

Consider the following result [11].

Theorem 3.2. *Let D be an operator of Laplace type on the space of sections $C^\infty(V)$ to a vector bundle V over a compact manifold M. Let I be the identity endomorphism of V. We have*

$$\begin{aligned} a_0(D) &= (4\pi)^{-m/2}\int_M Tr\{I\}, \qquad (3.19) \\ a_2(D) &= (4\pi)^{-m/2}\frac{1}{6}\int_M Tr(6E + \tau I), \\ a_4(D) &= (4\pi)^{-m/2}\frac{1}{360}\int_M Tr\{60E + 60\tau E + 180E^2 + 30\Omega^2 + (12\tau + 5\tau^2 - 2|\rho|^2 + \\ &+ 2|R|^2)I\}. \end{aligned}$$

Now we will extend Theorem 1.1 for s-manifolds with proof.

Theorem 3.3. *Let $M_i = (M_i, \eta_i, g_i, f_i, \xi_i)$ be m_i-dimensional compact S-manifolds without boundary with $m_i \geq 5$. Assume that $Spec(\triangle_p, M_1) = Spec(\triangle_p, M_2)$ for $p = 0, 1, 2$, then:*

(1) $m_1 = m_2$ and $Vol(M_1) = Vol(M_2)$.
(2) M_1 has constant scalar curvature k if and only if the manifold M_2 has constant scalar curvature k.
(3) M_1 is η-Einstein if and only if M_2 is η-Einstein.
(4) M_1 is S-space form with constant f-sectional curvature k if and only if M_2 is S-space form with constant f-sectional curvature k.

The values p=0, 1, 2 are not particularly special.

Proof. Let $M(\eta, g, f, \xi)$ be a $(2m+s)$-dimensional compact S-manifold without boundary. From (3.2) Theorem for $D = \triangle_p$(p=0, 1, 2), we have

$$\begin{aligned} Tr_{L^2}(e^{-t}\triangle_0) &= (4\pi t)^{-m/2}\{Vol(M) + O(t)\}, \\ a_2(\triangle_0, M) &= \frac{1}{6}(4\pi)^{-m/2}\int_M \tau, \\ a_2(\triangle_1, M) &= \frac{1}{6}(4\pi)^{-m/2}\int_M (2m+s)\tau. \end{aligned} \quad (3.20)$$

The work of Patodi shows that there exist universal constants so:

$$a_4(\triangle_p, M) = (4\pi)^{-m/2}\int_M \{c^1_{m,p}\tau^2 + c^2_{m,p}\rho^2 + c^1_{m,p}R^2 + c^1_{m,p}\tau\}, \quad p = 0, 1, 2. \quad (3.21)$$

Let $M_i = (M_i, \eta_i, g_i, f_i, \xi_i)$ be $m_i + s$-dimensional compact S-manifolds without boundary (i = 1, 2). Assume that $Spec(\triangle_p, M_1) = Spec(\triangle_p, M_2)$ for p=0, 1, 2. Let R_i, ρ_i and τ_i denote the curvature tensor, the Ricci tensor and the scalar curvature of $M_i (i = 1, 2)$ respectively. Then by Theorem 3.2(1) we have

$m_1 = m_2$ and $Vol(M_1) = Vol(M_2)$.
Also

$$\begin{aligned} \tau_1 &= (4\pi)^{m/2} Vol(M_1)^{-1}\{ma_2(\triangle_0, M_1) - a_2(\triangle_1, M_1)\} \\ &= (4\pi)^{m/2} Vol(M_2)^{-1}\{ma_2(\triangle_0, M_2) - a_2(\triangle_1, M_2)\} \\ &= \tau_2. \end{aligned} \quad (3.22)$$

Further suppose that M_1 is an η-Einstein S-manifold with coefficient functions α_1 and β_1. Here α_1 and β_1 are constant and hence the scalar curvature τ_1 of M_1 is also constant given as $\tau_1 = (2m+s)\alpha_1 + \beta_1$. Thus from second assertion of theorem the scalar curvature τ_2 of M_2 is also constant and $\tau_1 = \tau_2$. Since $Vol(M_1) = Vol(M_2)$, the integrals of τ^2 are equal. Since $\tau_{ii} = 0$, from a_4, we have

$$\int_{M_1} (c^2_{m,p}\rho_1^2 + c^3_{m,p}R_1^2) = \int_{M_2} (c^2_{m,p}\rho_2^2 + c^3_{m,p}R_2^2), \quad (3.23)$$

for $p = 1, 2$ these two equations are independent [4]. Consequently

$$\int_{M_1} \rho_1^2 = \int_{M_2} \rho_2^2, \quad \int_{M_1} R_1^2 = \int_{M_2} R_2^2. \quad (3.24)$$

Thus from (3.7), we have

$$0 = \int_{M_1} |S^1_{a_1,b_1}|^2 = \int_{M_1} |\rho_1|^2 + (2m+s)a_1^2 + sb_1^2 - 2a\tau_1 + 2a_1b_1s - 4b_1ms \quad (3.25)$$

$$= \int_{M_2} |\rho_2|^2 + (2m+s)a_1^2 + sb_1^2 - 2a_1\tau_1 + 2a_1b_1s - 4b_1ms. \quad (3.26)$$

Here we may note that

$$a_1 = \frac{\tau_1}{2m} - s = \frac{\tau_2}{2m} - s \quad (3.27)$$

$$b_1 = s(2m+s) - \frac{s\tau_1}{2m} = s(2m+s) - \frac{s\tau_2}{2m}. \quad (3.28)$$

Here we may take

$$S^2_{a,b} = \rho_2 - (a_2 g_2 + b_2 \sum_{\alpha=1}^{s} \eta_{\alpha 2} \otimes \eta_{\alpha 2}), \quad (3.29)$$

where

$$a_2 = \frac{\tau_2}{2m} - s, \quad b_2 = s(2m+s) - \frac{s\tau_2}{2m}, \tag{3.30}$$

then we have

$$\int_{M_2} |S^2_{a_2,b_2}|^2 = \int_{M_2} |\rho_2|^2 + (2m+s)a_2^2 + sb_2^2 - 2a_2\tau_2 + 2a_2b_2s - 4b_2ms. \tag{3.31}$$

This implies

$$a_1 = a_2, \quad b_1 = b_2. \tag{3.32}$$

Therefore all above equation implies

$$0 = \int_{M_2} |S^2_{a_2,b_2}|^2, \tag{3.33}$$

then M_2 is an η-Einstein manifold with the same coefficients in the defining equation. This completes the proof of Theorem 3.3 (3). Lastly, suppose that M_1 is s-space form with constant f-sectional curvature k. Then from (3.1), M_1 is η-Einstein with constant coeficients $a_1 = \frac{4s+(k+3s)(2m-1)+3(k-1)}{4}$ and $b_1 = \frac{(2m+s-2)(4-k-3s)-3(k-s)}{4}$. Thus the scalar cutvature is

$$\tau_1 = \frac{m}{2}\{(2m+2)k + 6m + 8s - 6\}. \tag{3.34}$$

Therefore from assertion (3) and hypothesis that $Spec(\triangle_p, M_1) = Spec(\triangle_p, M_2)(p = 0, 1, 2)$, we see that M_2 is η-Einstein manifold with constant coefficients a_2 and b_2 such that $a_2 = a_1$, $b_2 = b_1$ and hence $\tau_2 = \tau_1$. Now we define the tensor field for the S-manifold M_2 as:

$$(T_c^2)_{ijkl} = R_{ijkl} - K_{ijkl}, \tag{3.35}$$

where

$$\begin{aligned} K_{ijkl} = \sum_{\alpha,\beta} \{ & g(fe_i, e_l)\eta_\alpha(e_j)\eta_\beta(e_k) - g(fe_i, e_k)\eta_\alpha(e_j)\eta_\beta(e_l) + \\ & + g(fe_j, fe_k)\eta_\alpha(e_i)\eta_\beta(e_l) - g(fe_j, fe_l)\eta_\alpha(e_i)\eta_\beta(e_k)\} + \\ & + \frac{1}{4}(k+3s)\{g(fe_i, fe_l)g(fe_j, fe_k) - g(fe_i, fe_k)g(fe_j, fe_l)\} + \\ & + \frac{1}{4}(k-s)\{F(e_i, e_l)F(e_j, e_k) - F(e_i, e_k)F(e_j, e_l) - 2F(e_i, e_j)F(e_k, e_l)\}, \end{aligned} \tag{3.36}$$

after some computations, we have

$$|K|^2 = m\{(2m+2)k^2 + 6m + 8s - 6\}. \tag{3.37}$$

Now using lemma

$$R_{ijkl}K_{ijkl} = 2k\tau - m(k-s)(6m - 6 + 8s), \tag{3.38}$$

from (3.35), (3.37) and (3.38), we obtain

$$|T_c^2|^2 = |R_2|^2 - 4k\tau + m\{(2m+2)k^2 + 6m + 8s - 6\} + 2m(k-s)(6m - 6 + 8s). \tag{3.39}$$

Also, since M_1 is m-dimensional S-space form with constant f-sectional curvature k, we have

$$|R_1|^2 = m\{(2m+2)k^2 + 6m + 8s - 6\}, \tag{3.40}$$

and

$$0 = |T_c^1|^2 = |R_1|^2 - 4k\tau_1 + m\{(2m+2)k^2 + 6m + 8s - 6\} + 2m(k-s)(6m - 6 + 8s). \quad (3.41)$$

Thus from (3.24), (3.39), (3.41) and $\tau_1 = \tau_2$ we obtain

$$\begin{aligned} 0 &= \int_{M_1} |T_c^1|^2 \\ &= \int_{M_1} |R_1|^2 - 4k\tau_1 + m\{(2m+2)k^2 + 6m + 8s - 6\} + 2m(k-s)(6m - 6 + 8s) \\ &= \int_{M_2} |R_2|^2 - 4k\tau_2 + m\{(2m+2)k^2 + 6m + 8s - 6\} + 2m(k-s)(6m - 6 + 8s) \\ &= \int_{M_2} |T_c^2|^2, \end{aligned} \quad (3.42)$$

and hence $T_c^2 = 0$ on M_2. Therefore we see that M_2 is also an $(2m+s)-$dimensional S- space form with constant $f-$sectional curvature k. This completes the proof of Theorem.

4 Conclusion

In this paper, We have examined the spectral geometry of η-Einstein S-manifolds and have computed the spectral coefficients for S-space form to obtain the results analogue to Patodi's results for Riemannian manifolds and J. Park results for $\eta-$Einstein Sasakian manifolds. It is showed that how an $\eta-$Einstein S-manifold and S-space forms are spectrally determined.

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Competing Interests

Author has declared that no competing interests exist.

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