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The Numerical Study of Efforts from the 4r Symmetrical Spherical Quadrilateral Mechanism

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Author's contribution

The sole author designed, analyzed and interpreted and prepared the manuscript.

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ABSTRACT

The 4r spherical quadrilateral mechanism is made of three mobile elements and one fixed (the basis) connected through the four revolute kinematic pairs, whose axes intersect at the same point. It is multiple statically indeterminate and for calculating the efforts from kinematic pairs it is using the elastic linear calculation, in which the mechanism elements are considered deformabile rigid. The displacements in the local systems of reference, produced by external requests are small and defined by the vectors of small rotations, which act similarly to the angular velocity and small displacements, defined by displacement vectors. The use of the relative displacements method, with the expression of displacements under the form of a column matrix in plukeriane coordinates, makes possible the calculation of efforts and reactions in the kinematic pairs of this mechanism. The results of the numerical solving of this problem will be presented under the form of a diagrams and will be commented.

Keywords: Quadrilateral mechanism; elastic calculation; cardan joint.

1. INTRODUCTION

This mechanism presents interest because it reproduces, cinematically speaking, the movement of a system with a cardan joint [1,2]. Being statically undetermined [3,4], for the 4R symmetrical spherical quadrilateral mechanism is necessary to be used the elastic linear

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calculation [5,6], for calculating the reactions. In mechanism, the elements are found in different systems of reference [7,8]. The equations of elastic balanced are written in the chosen general system of reference [9,10]. The physical type of these reactions is perceived by expressing them in the local reference systems. The reactions will be calculated and in local references systems, will be expressed under the form of a diagrams and will be commented. The obtained results can lead to conclusions referring to the influence of geometrical and mechanical parameters over the unitary efforts that appear in the kinematic pairs of the mechanism [11,12].

2. MATERIALS AND METHODS

The mechanically speaking (static), the 4R mechanism has unknown the reactions from the kinematic pairs A, 2, 3, 5, 6, 8 and also the moment from the joint A (see Fig. 1), in total 6+5x5=31 unknowns and 3 elements x 6=18 equations, 31-18=13 times statically undetermined [7,8].

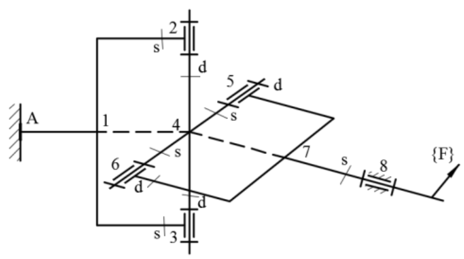

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Fig. 1. The 4r symmetrical spherical quadrilateral mechanism

For determining these components a linear elastic calculation is used. In the following, a mathematical model with be elaborated for the linear elastic calculation of reactions, model that has as basis the method of relative displacements, presented in paper [13]., with the notation in pluckerian coordinates.

2.1 Mathematical Model

In the elastic calculation the joint from A is blocked (see Fig. 1), thing that explains the apparition as unknown of the axial moment in that point. The pivotal points 1, 4, 7, have the displacements $\{\Delta_1\}$, $\{\Delta_4\}$, $\{\Delta_7\}$, and the pivotal points with the cinematic couples 2, 3, 5, 6 have left-right displacements with $\{\Delta_2^s\}$, $\{\Delta_2^d\}$, $\{\Delta_3^s\}$, $\{\Delta_3^d\}$, $\{\Delta_5^s\}$, $\{\Delta_5^d\}$, $\{\Delta_6^s\}$, $\{\Delta_6^d\}$. Pivotal point 8 has the displacement to left $\{\Delta_8^s\}$ and to right $\{\Delta_8^d\}=\{0\}$. So the relations are written under the form:

$$
\begin{aligned}
\{\Delta_2^d\} &= \{\Delta_2^s\}+\xi_2\{U_2\}\\
\{\Delta_3^d\} &= \{\Delta_3^s\}+\xi_3\{U_3\}\\
\{\Delta_5^d\} &= \{\Delta_5^s\}+\xi_5\{U_5\}\\
\{\Delta_6^d\} &= \{\Delta_6^s\}+\xi_6\{U_6\}\\
\{0\} &= \{\Delta_8^s\}+\xi_7\{U_7\}.
\end{aligned}
\tag{1}
$$

From the pivotal points balance $[Kij]=[Kji]$.

$$
\begin{cases}
[K_{12}]\left(\{\Delta_1\}-\{\Delta_2^s\}\right)+[K_{1A}]\left(\{\Delta_1\}-\{\Delta_A^0\}\right)+[K_{13}]\left(\{\Delta_1\}-\{\Delta_3^s\}\right)=\{0\}\\
[K_{21}]\left(\{\Delta_2^s\}-\{\Delta_1\}\right)+[K_{24}]\left(\{\Delta_2^d\}-\{\Delta_4\}\right)=\{0\}\\
[K_{31}]\left(\{\Delta_3^s\}-\{\Delta_1\}\right)+[K_{34}]\left(\{\Delta_3^d\}-\{\Delta_4\}\right)=\{0\}\\
[K_{42}]\left(\{\Delta_4\}-\{\Delta_2^d\}\right)+[K_{43}]\left(\{\Delta_4\}-\{\Delta_3^d\}\right)+[K_{45}]\left(\{\Delta_4\}-\{\Delta_5^s\}\right)+[K_{46}]\left(\{\Delta_4\}-\{\Delta_6^s\}\right)=\{0\}\\
[K_{54}]\left(\{\Delta_5^s\}-\{\Delta_4\}\right)+[K_{57}]\left(\{\Delta_5^d\}-\{\Delta_7\}\right)=\{0\}\\
[K_{64}]\left(\{\Delta_6^s\}-\{\Delta_4\}\right)+[K_{67}]\left(\{\Delta_6^d\}-\{\Delta_7\}\right)=\{0\}\\
[K_{75}]\left(\{\Delta_7\}-\{\Delta_5^d\}\right)+[K_{76}]\left(\{\Delta_7\}-\{\Delta_6^d\}\right)+[K_{78}]\left(\{\Delta_7\}-\{\Delta_8^s\}\right)=\{0\}.
\end{cases}
\tag{2}
$$

Are noted

$$
\begin{aligned}
&[K_{11}]=[K_{1A}]+[K_{12}]+[K_{13}]\\
&[K_{22}]=[K_{21}]+[K_{24}]\ ;\ [K_{33}]=[K_{31}]+[K_{34}]\\
&[K_{44}]=[K_{42}]+[K_{43}]+[K_{45}]+[K_{46}]\\
&[K_{55}]=[K_{54}]+[K_{57}]\ ;\ [K_{66}]=[K_{64}]+[K_{67}]\\
&[K_{77}]=[K_{75}]+[K_{76}]+[K_{78}].
\end{aligned}
\tag{3}
$$

and taking into account the relations (1) it results

$$\begin{cases}[K_{11}]\{\varDelta_1\}-[K_{12}]\{\varDelta_2^s\}-[K_{13}]\{\varDelta_3^s\}=\{0\}\\ [K_{21}]\{\varDelta_1\}+[K_{22}]\{\varDelta_2^s\}-[K_{24}]\{\varDelta_4\}+\xi_2[K_{24}]\{U_2\}=\{0\}\\ -[K_{31}]\{\varDelta_1\}+[K_{33}]\{\varDelta_3^s\}-[K_{34}]\{\varDelta_4\}+\xi_3[K_{34}]\{U_3\}=\{0\}\\ [K_{31}]\{\varDelta_1\}+[K_{33}]\{\varDelta_3^s\}-[K_{34}]\{\varDelta_4\}+\xi_3[K_{34}]\{U_3\}=\{0\}\\ -[K_{54}]\{\varDelta_4\}-[K_{55}]\{\varDelta_5^s\}-[K_{57}]\{\varDelta_7\}+\xi_5[K_{57}]\{U_5\}=\{0\}\\ -[K_{64}]\{\varDelta_4\}+[K_{66}]\{\varDelta_6^s\}-[K_{67}]\{\varDelta_7\}+\xi_6[K_{67}]\{U_6\}=\{0\}\\ [K_{75}]\{\varDelta_5^s\}-[K_{76}]\{\varDelta_6^s\}+[K_{77}]\{\varDelta_7\}-\xi_5[K_{75}]\{U_5\}-\xi_6[K_{76}]\{U_6\}+\xi_8[K_{78}]\{U_8\}=\{0\}\end{cases}. \tag{4}$$

The notations are made as

$$\begin{aligned}&\{\varDelta\}=\left[\{\varDelta_1\}^T,\{\varDelta_2^s\}^T,\{\varDelta_3^s\}^T,\{\varDelta_4\}^T,\{\varDelta_5^s\}^T,\{\varDelta_6^s\}^T,\{\varDelta_7\}^T\right]^T\\ &\{\xi\}=[\xi_2,\xi_3,\xi_5,\xi_6,\xi_8]\\ &[V_2]=[1,0,0,0,0]\ ;\ [V_3]=[0,1,0,0,0]\ ;\ [V_5]=[0,0,1,0,0]\\ &[V_6]=[0,0,0,1,0]\ ;\ [V_8]=[0,0,0,0,1].\end{aligned} \tag{5}$$

and then

$$\zeta_i=[V_i]\{\xi\}. \tag{6}$$

and $\zeta_2[K_{24}]\{U_2\}=[K_{24}]\{U_2\}[V_2]\{\xi\}$, and the analogue.

Also, the notations are made

$$[K_1]=\begin{bmatrix}[K_{11}] & -[K_{12}] & -[K_{13}] & [0] & [0] & [0] & [0]\\ -[K_{21}] & [K_{22}] & [0] & -[K_{24}] & [0] & [0] & [0]\\ -[K_{31}] & [0] & [K_{33}] & [K_{34}] & [0] & [0] & [0]\\ [0] & -[K_{42}] & -[K_{43}] & [K_{44}] & -[K_{45}] & -[K_{46}] & [0]\\ [0] & [0] & [0] & -[K_{54}] & [K_{55}] & [0] & -[K_{57}]\\ [0] & [0] & [0] & -[K_{64}] & [0] & [K_{66}] & -[K_{67}]\\ [0] & [0] & [0] & [0] & -[K_{75}] & -[K_{76}] & [K_{77}]\end{bmatrix}. \tag{7}$$

$$[K_2]=\begin{bmatrix}[0]\\ [K_{24}]\{U_2\}[V_2]\\ [K_{34}]\{U_3\}[V_3]\\ -[K_{24}]\{U_2\}[V_2]-[K_{34}]\{U_3\}[V_3]\\ [K_{57}]\{U_5\}[V_5]\\ [K_{67}]\{U_6\}[V_6]\\ -[K_{57}]\{U_5\}[V_5]-[K_{67}]\{U_6\}[V_6]+[K_{78}]\{U_8\}[V_8]\end{bmatrix}. \tag{8}$$

and then the equations (4) are combined into the equation

$$[K_1]\{\Delta\}+[K_2]\{\xi\}=\{0\}. \tag{9}$$

equivalent with 42 scalar equations.

Isolating the left side of the pair 2, (see Fig. 1) results that

$$\{R_2\}=\{E_2^s\}=[K_{21}]\left\{\{\Delta_2^s\}-\{\Delta_1\}\right\}. \tag{10}$$

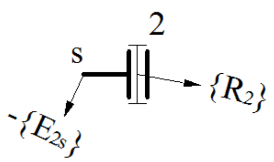


Fig. 2. The isolation of kinematic 2

and the analogue

$$\begin{aligned}
\{R_3\}&=\{E_3^s\}=[K_{31}]\left\{\{\Delta_3^s\}-\{\Delta_1\}\right\}\\
\{R_5\}&=\{E_5^s\}=[K_{54}]\left\{\{\Delta_5^s\}-\{\Delta_4\}\right\}\\
\{R_6\}&=\{E_6^s\}=[K_{64}]\left\{\{\Delta_6^s\}-\{\Delta_4\}\right\}\\
\{R_8\}&=\{E_8^s\}=\{E\}=[K_{87}]\left\{\{\Delta_8^s\}-\{\Delta_7\}\right\}.
\end{aligned} \tag{11}$$

as [13], $\{\tilde{U}_i\}^T\{R_i\}=0$ and $\{\Delta_8^s\}=-\xi_8\{U_8\}$ the equations are obtained

$$\begin{cases}
\{\tilde{U}_2\}^T[K_{21}]\{\Delta_2^s\}-\{\tilde{U}_2\}^T[K_{21}]\{\Delta_1\}=0\\
\{\tilde{U}_3\}^T[K_{31}]\{\Delta_3^s\}-\{\tilde{U}_3\}^T[K_{31}]\{\Delta_1\}=0\\
\{\tilde{U}_5\}^T[K_{54}]\{\Delta_5^s\}-\{\tilde{U}_5\}^T[K_{54}]\{\Delta_4\}=0\\
\{\tilde{U}_6\}^T[K_{64}]\{\Delta_6^s\}-\{\tilde{U}_6\}^T[K_{64}]\{\Delta_4\}=0\\
\{\tilde{U}_8\}^T[K_{87}]\{\Delta_7\}+\xi_8\{\tilde{U}_8\}^T[K_{87}]\{U_8\}=\{\tilde{U}_8\}^T[F]
\end{cases}. \tag{12}$$

With the notations

$$[K_3]=\begin{bmatrix}
\{\tilde{U}_2\}^T[K_{21}] & -\{\tilde{U}_2\}^T[K_{21}] & 0 & 0 & 0 & 0 & 0\\
\{\tilde{U}_3\}^T[K_{31}] & 0 & -\{\tilde{U}_3\}^T[K_{31}] & 0 & 0 & 0 & 0\\
0 & 0 & 0 & \{\tilde{U}_5\}^T[K_{54}] & -\{\tilde{U}_5\}^T[K_{54}] & 0 & 0\\
0 & 0 & 0 & \{\tilde{U}_6\}^T[K_{64}] & 0 & -\{\tilde{U}_6\}^T[K_{64}] & 0\\
0 & 0 & 0 & 0 & 0 & 0 & \{\tilde{U}_8\}^T[K_{78}]
\end{bmatrix} \tag{13}$$

$$[K_4]=\begin{bmatrix}
0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & \{\tilde{U}_8\}^T[K_{78}]\{U_8\}
\end{bmatrix}\ ;\ \{F\}=\begin{bmatrix}0\\0\\0\\0\\\{\tilde{U}_8\}^T[F]\end{bmatrix}. \tag{14}$$

the equations (11) are combined in the matrix equation

$$[K_3]\{\Delta\}+[K_4]\{\xi\}=\{F\}. \tag{15}$$

equivalent with 5 scalar equations.

The equations (7), (15) can be narrowed with the notations

$$[K]=\begin{bmatrix}[K_1]_1 & [K_2]\\ [K_3] & [K_4]\end{bmatrix}. \tag{16}$$

in the equation

$$[K]\begin{bmatrix}\{\Delta\}\\ \{\xi\}\end{bmatrix}=\begin{bmatrix}\{0\}\\ \{F\}\end{bmatrix}. \tag{17}$$

equivalent with 47 equations with 47 unknowns from which results $\{\Delta\}$ and $\{\xi\}$, and the reactions and efforts are calculated with the relations (10), (11) in which $\{\Delta_8^s\} = -\xi_8\{U_8\}$.

2.2 Calculation Algorithm

1. The indexation of bars is done from $1,2,...,14$ and the lengths are noted with l_i, $1,2,...,14$ and $l_2 = l_4$; $l_3 = l_5$; $l_6 = l_7$; $l_8 = l_9$; $l_{10} = l_{12}$; $l_{11} = l_{13}$.

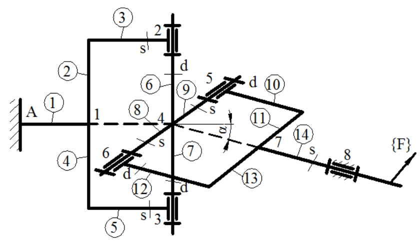


Fig. 3. Indexing bars

2. The inertia moment is calculated:

$$I_{iy} = I_{iz} = \frac{\pi d_i^4}{64}\ ;\ I_{ix} = I_{iy} + I_{iz}\ ;\ A_i = \frac{\pi d_i^2}{4}. \quad (18)$$

3. The rigidity matrixes are calculated in the local reference system:

$$[k_i] = \begin{bmatrix} 0 & 0 & 0 & \frac{EA}{l} & 0 & 0 \\ 0 & 0 & \frac{6EI_{iz}}{l^2} & 0 & \frac{12EI_{iz}}{l^3} & 0 \\ 0 & -\frac{6EI_{iy}}{l^2} & 0 & 0 & 0 & \frac{12EI_{iy}}{l^3} \\ \frac{GI_{ix}}{l} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{4EI_{iy}}{l} & 0 & 0 & 0 & -\frac{6EI_{iy}}{l^2} \\ 0 & 0 & \frac{4EI_{iz}}{l} & 0 & \frac{6EI_{iz}}{l^2} & 0 \end{bmatrix}. \quad (19)$$

$$[h_i] = \begin{bmatrix} 0 & 0 & 0 & \frac{l}{GI_{ix}} & 0 & 0 \\ 0 & 0 & \frac{l^2}{2EI_{iy}} & 0 & \frac{l}{EI_{iy}} & 0 \\ 0 & -\frac{l^2}{2EI_{iz}} & 0 & 0 & 0 & \frac{l}{EI_{iz}} \\ \frac{l}{EA} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{l^3}{3EI_{iz}} & 0 & 0 & 0 & -\frac{l^2}{2EI_{iz}} \\ 0 & 0 & \frac{l^3}{3EI_{iy}} & 0 & \frac{l^2}{2EI_{iy}} & 0 \end{bmatrix}. \quad (20)$$

4. The matrix $[G_i];[R_i]$ are calculated, with the relations from (Table 1) and then the product $[G_i][R_i]$

$$[G_i]=\begin{bmatrix} 0 & -Z_i & Y_i \\ Z_i & 0 & -X_i \\ -Y_i & X_i & 0 \end{bmatrix}. \tag{21}$$

5. The position matrixes are calculated:

$$[T_i]=\begin{bmatrix} [R_i] & [0] \\ [G_i][R_i] & [R_i] \end{bmatrix} \ ; \ [T_i]^{-1}=\begin{bmatrix} [R_i]^T & [0] \\ [R_i]^T[G_i]^T & [R_i]^T \end{bmatrix}. \tag{22}$$

6. The matrixes $[H_i^*],[K_i^*]$ are calculated:

$$[H_i^*]=[T_i][h_i][T_i]^{-1} \ ; \ [K_i^*]=[T_i][k_i][T_i]^{-1} \tag{23}$$

7. The matrixes $[H_{23}^*],[H_{45}^*],[H_{10,11}^*],[H_{12,13}^*]$ are calculated:

$$\begin{gathered} [H_{23}^*]=[H_2^*]+[H_3^*] \ ; \ [H_{45}^*]=[H_4^*]+[H_5^*] \\ [H_{10,11}^*]=[H_{10}^*]+[H_{11}^*] \ ; \ [H_{12,13}^*]=[H_{12}^*]+[H_{13}^*]. \end{gathered} \tag{24}$$

8. It is identified:

$$\begin{gathered} [K_{1A}^*]=[K_{11}^*] \ ; \ [K_{12}^*]=[H_{23}^*]^{-1}; \ [K_{13}^*]=[H_{45}^*]^{-1} \\ [K_{24}^*]=[K_6^*] \ ; \ [K_{34}^*]=[K_7^*] \ ; \ [K_{45}^*]=[K_9^*] \\ [K_{46}^*]=[K_8^*] \ ; \ [K_{57}^*]=[H_{10,11}^*]^{-1}; \ [K_{67}^*]=[H_{12,13}^*]^{-1} \\ [K_{67}^*]=[H_{12,13}^*]^{-1}; \ [K_{78}^*]=[K_{14}^*]. \end{gathered} \tag{25}$$

9. θ_2 is calculated with the formulas:

$$\theta_2=\begin{cases} arctg(\frac{1}{c\alpha}tg\theta_1);0\le\theta_1<\frac{\pi}{2} \\ \frac{\pi}{2};\theta_1=\frac{\pi}{2} \\ \pi+arctg(\frac{1}{c\alpha}tg\theta_1);\frac{\pi}{2}\le\theta_1<\frac{3\pi}{2} \\ \frac{3\pi}{2};\theta_1=\frac{3\pi}{2} \\ 2\pi+arctg(\frac{1}{c\alpha}tg\theta_1);\frac{3\pi}{2}<\theta_1\le 2\pi \end{cases}. \tag{26}$$

10. $[T_{AB}],[T_{BC}],[T_{CD}]$ are calculated with the formulas:

$$[T_{AB}]=\begin{bmatrix}[R_{AB}] & [0]\\ [0] & [R_{AB}]\end{bmatrix} \quad ; \quad [T_{BC}]=\begin{bmatrix}[R_{BC}] & [0]\\ [0] & [R_{BC}]\end{bmatrix}$$
$$[T_{CD}]=\begin{bmatrix}[R_{CD}] & [0]\\ [0] & [R_{CD}]\end{bmatrix}. \tag{27}$$

where

$$[R_{AB}]=\begin{bmatrix}1 & 0 & 0\\ 0 & c\theta_1 & -s\theta_1\\ 0 & s\theta_1 & c\theta_1\end{bmatrix} \quad ; \quad [R_{BC}]=\begin{bmatrix}c\theta_1 c\theta_2 + s\theta_1 s\theta_2 c\alpha & s\alpha s\theta_2 & 0\\ -c\theta_1 s\theta_2 s\alpha & c\theta_2 & -s\theta_1\\ -s\theta_1 s\theta_2 s\alpha & c\alpha s\theta_2 & c\theta_1\end{bmatrix}$$
$$[R_{CD}]=\begin{bmatrix}c\alpha & s\alpha s\theta_2 & s\alpha c\theta_2\\ 0 & c\theta_2 & -s\theta_2\\ -s\alpha & c\alpha s\theta_2 & c\alpha c\theta_2\end{bmatrix}. \tag{28}$$

11. Are calculated:

$[K_{1A}],[K_{12}],[K_{13}]$ with the formula

$$[K]=[T_{AB}][K^*][T_{AB}]^{-1}. \tag{29}$$

$[K_{24}],[K_{34}],[K_{45}],[K_{46}]$ with formulas type

$$[K]=[T_{BC}][K^*][T_{BC}]^{-1}. \tag{30}$$

$[K_{57}],[K_{67}],[K_{78}]$ with formulas type

$$[K]=[T_{CD}][K^*][T_{CD}]^{-1}. \tag{31}$$

12. Are calculated:

$$\{U_2\}=\{U_3\}=[0,-s\theta_1,c\theta_1,0,0,0]^T$$
$$\{U_5\}=\{U_6\}=[s\theta_2 s\alpha,c\theta_2,s\theta_2 c\alpha,0,0,0]^T \tag{32}$$
$$\{U_8\}=[c\alpha,0,-s\alpha,0,0,0]^T.$$

$$\{\tilde{U}_2\}=\{\tilde{U}_3\}=[0,0,0,0,-s\theta_1,c\theta_1]^T$$
$$\{\tilde{U}_5\}=\{\tilde{U}_6\}=[0,0,0,s\theta_2 s\alpha,c\theta_2,s\theta_2 c\alpha]^T \tag{33}$$
$$\{\tilde{U}_8\}=[0,0,0,c\alpha,0,-s\alpha]^T.$$

$$\{F\}=\tilde{M}[0,0,0,c\alpha,0,-s\alpha]^T \quad ; \quad \{\tilde{U}_8\}\{F\}=\tilde{M}. \tag{34}$$

13. Are calculated the matrixes $[K_1],[K_2],[K_3],[K_4],\{F\},[K]$ with the formulas (7), (8), (13), (14), (16).
14. It is solved the matrix equation (17).
15. The reaction from A is calculated with the relation

$$\{R_A\}=-[K_{1A}]\{\Delta_1\}. \tag{35}$$

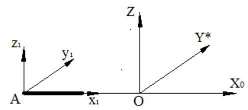
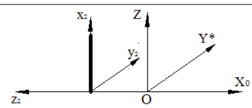
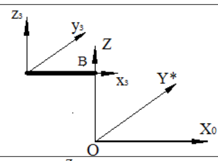
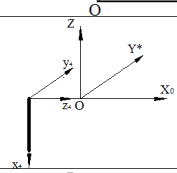
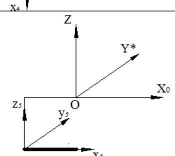
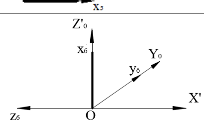
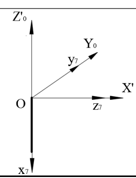
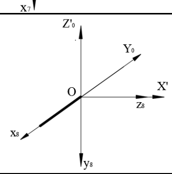
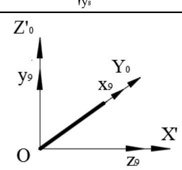
and expressed in local coordinates.

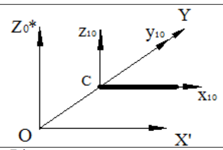
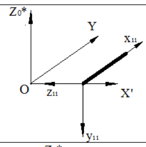
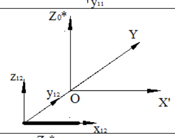
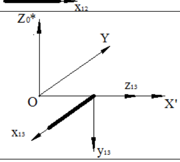
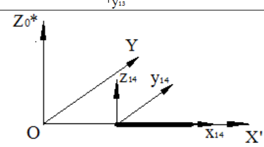
16. The reactions are calculated $\{R_2\},\{R_3\},\{R_5\},\{R_6\},\{R_8\}$ with the relations (10), (11).
17. The reactions are expressed under $\{R_2\},\{R_3\},\{R_5\},\{R_6\},\{R_8\}$ in local system coordinates.
18. The graphs

$$\xi_2,\xi_3,\xi_5,\xi_6,\xi_8,R_{1x},R_{1y},R_{1z},M_{1x},M_{1y},M_{1z},$$
$$R_{2x},R_{2y},R_{2z},M_{2x},M_{2y},M_{2z},$$
$$R_{3x},R_{3y},R_{3z},M_{3x},M_{3y},M_{3z},$$
$$R_{5x},R_{5y},R_{5z},M_{5x},M_{5y},M_{5z},$$
$$R_{6x},R_{6y},R_{6z},M_{6x},M_{6y},M_{6z},$$
$$R_{8x},R_{8y},R_{8z},M_{8x},M_{8y},M_{8z}.$$

are made taking into account θ_1.

Table 1. The espressions of the position and rotation matrixes $[G_i],[R_i]$

The element	The local reference system $Ox_iy_iz_i$	The coordinates of the local reference system X_i, Y_i, Z_i	The position matrix $[G_i]$	The rotation matrix $[R_i]$
1		$X_1 = -(l_1 + l_3)$ $Y_1 = 0$ $Z_1 = 0$	$[G_1]=\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & l_1+l_3 \\ 0 & -(l_1+l_3) & 0 \end{bmatrix}$	$[R_1]=\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
2		$X_2 = -l_3$ $Y_2 = 0$ $Z_1 = 0$	$[G_2]=\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & l_3 \\ 0 & -l_3 & 0 \end{bmatrix}$	$[R_2]=\begin{bmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$
3		$X_3 = -l_3$ $Y_3 = 0$ $Z_3 = l_2$	$[G_3]=\begin{bmatrix} 0 & -l_2 & 0 \\ l_2 & 0 & l_3 \\ 0 & -l_3 & 0 \end{bmatrix}$	$[R_3]=\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
4		$X_4 = -l_3$ $Y_4 = 0$ $Z_4 = 0$	$[G_4]=\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & l_3 \\ 0 & -l_3 & 0 \end{bmatrix}$	$[R_4]=\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{bmatrix}$
5		$X_5 = -l_3$ $Y_5 = 0$ $Z_5 = -l_4$	$[G_5]=\begin{bmatrix} 0 & l_4 & 0 \\ -l_4 & 0 & l_3 \\ 0 & -l_3 & 0 \end{bmatrix}$	$[R_5]=\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
6		$X_6 = 0$ $Y_6 = 0$ $Z_6 = 0$	$[G_6]=\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$	$[R_6]=\begin{bmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$
7		$X_7 = 0$ $Y_7 = 0$ $Z_7 = 0$	$[G_7]=\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$	$[R_7]=\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{bmatrix}$
8		$X_8 = 0$ $Y_8 = 0$ $Z_8 = 0$	$[G_8]=\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$	$[R_8]=\begin{bmatrix} 0 & 0 & 1 \\ -1 & 0 & 0 \\ 0 & -1 & 0 \end{bmatrix}$
9		$X_9 = 0$ $Y_9 = 0$ $Z_9 = 0$	$[G_9]=\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$	$[R_9]=\begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$

The element	**The local reference system** $Ox_i y_i z_i$	**The coordinates of the local reference system** X_i, Y_i, Z_i	**The position matrix** $[G_i]$	**The rotation matrix** $[R_i]$
10		$X_{10} = 0$ $Y_{10} = l_9$ $Z_{10} = 0$	$[G_{10}] = \begin{bmatrix} 0 & 0 & l_9 \\ 0 & 0 & 0 \\ -l_9 & 0 & 0 \end{bmatrix}$	$[R_{10}] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
11		$X_{11} = l_{10}$ $Y_{11} = 0$ $Z_{11} = 0$	$[G_{11}] = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -l_{10} \\ 0 & l_{10} & 0 \end{bmatrix}$	$[R_{11}] = \begin{bmatrix} 0 & 0 & -1 \\ 1 & 0 & 0 \\ 0 & -1 & 0 \end{bmatrix}$
12		$X_{12} = 0$ $Y_{12} = -l_8$ $Z_{12} = 0$	$[G_{12}] = \begin{bmatrix} 0 & 0 & -l_8 \\ 0 & 0 & 0 \\ l_8 & 0 & 0 \end{bmatrix}$	$[R_{12}] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
13		$X_{13} = l_{12}$ $Y_{13} = 0$ $Z_{13} = 0$	$[G_{13}] = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -l_{12} \\ 0 & l_{12} & 0 \end{bmatrix}$	$[R_{13}] = \begin{bmatrix} 0 & 0 & 1 \\ -1 & 0 & 0 \\ 0 & -1 & 0 \end{bmatrix}$
14		$X_{14} = l_{10}$ $Y_{14} = 0$ $Z_{14} = 0$	$[G_{14}] = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -l_{10} \\ 0 & l_{10} & 0 \end{bmatrix}$	$[R_{14}] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

3. RESULTS AND DISCUSSION

It is considered the 4r symmetrical spherical quadrilateral mechanism [1,8], from Fig. 3. Elements of the mechanism have these geometrical and mechanical parameters: the lengths $l_i = l = 0{,}05m$, the diameters $d_i = d = 0{,}02m$, the sections A_i, the elasticity modules $E = 2{,}1 \cdot 10^{11} N/m^2$, $G = 8{,}1 \cdot 10^{10} N/m^2$, the sections A_i and the main central inertial moments I_{iy}, I_{iz}, $I_{ix} = I_{iy} + I_{iz}$, $i = 1{,}2{,}...{,}14$. The mechanism is driven by torque $\widetilde{M} = 1Nm$.

The solving the mathematical model can be done using any conventional method of calculation numerical [14,15]. Basis of the mathematical model and the algorithm presented in this paper has been realized a calculation program of the reactions from the kinematic pairs of the mechanism. The program was realized in Excel and the following results were obtained. The reactions from the kinematic pair A, $R^0_{AX}, R^0_{AY}, R^0_{AZ}$ and for the moments $M^0_{AX}, M^0_{AY}, M^0_{AZ}$, in the case where $\alpha = 0^\circ$, the results from diagrams from Fig. 4(a). and Fig. 5(a). In local reference systems the reaction $R_{AX}, R_{AY}, R_{AZ}, M_{AX}, M_{AY}, M_{AZ}$, varies as shown in Fig. 4(b) and Fig. 5(b).

From the variation diagrams it is found that the reaction forces and moments are constant in the local systems of reference for all kinematic pairs. For the reaction from the joints 2, in general reference systems, are obtained the values presented in Fig. 6(a). and Fig. 7(a). and in own reference systems values presented in Fig. 6(b). and Fig. 7(b).

For the case where $\alpha = 10^\circ$ for the same kinematic pair A, are obtained the reactions $R^0_{AX}, R^0_{AY}, R^0_{AZ}$, $M^0_{AX}, M^0_{AY}, M^0_{AZ}$, who varies as shown in Fig. 8 and Fig. 9.

It is noticed that, the components $R^0_{AX}, R^0_{AY}, R^0_{AZ}$, $M^0_{AX}, M^0_{AY}, M^0_{AZ}$, in the local reference system aren't constant. Theirs variation depends on the angle θ_1 .In Fig. 10. and Fig. 11. are represented the variation of the components $R_{AX}^0, R_{AY}^0, R_{AZ}^0$, $M_{AX}^0, M_{AY}^0, M_{AZ}^0$,for $\alpha = 20^\circ, \alpha = 30^\circ$.

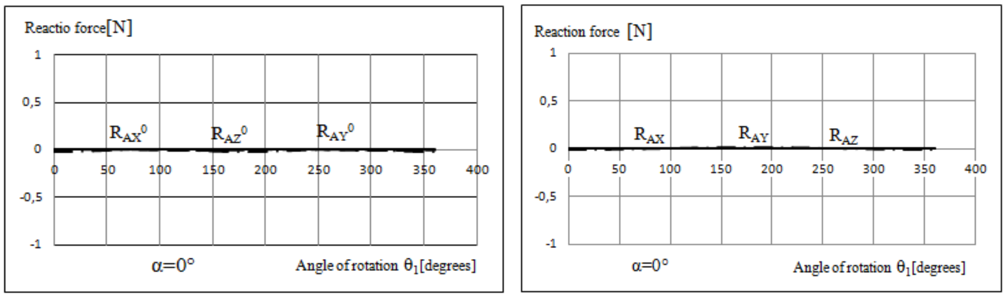


Fig. 4. The variation diagrams of reaction forces in joint A, for *α=0°* a) in general system of reference b) in local system oo reference

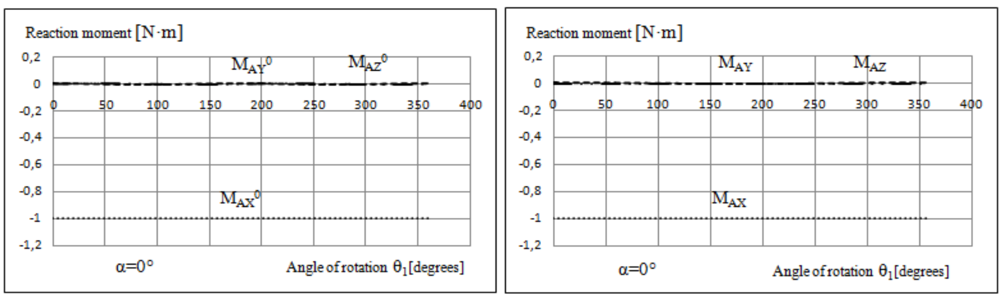


Fig. 5. The variation diagrams of reaction moments in joint A, for *α=0°* a) in general system of reference b) in local system of reference

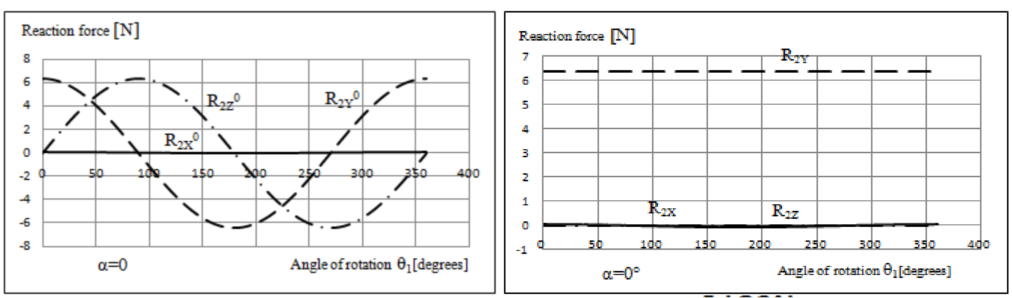


Fig. 6. The variation diagrams of reaction forces in joint 2, for *α=0°* a) in general system of reference b) in local system of reference

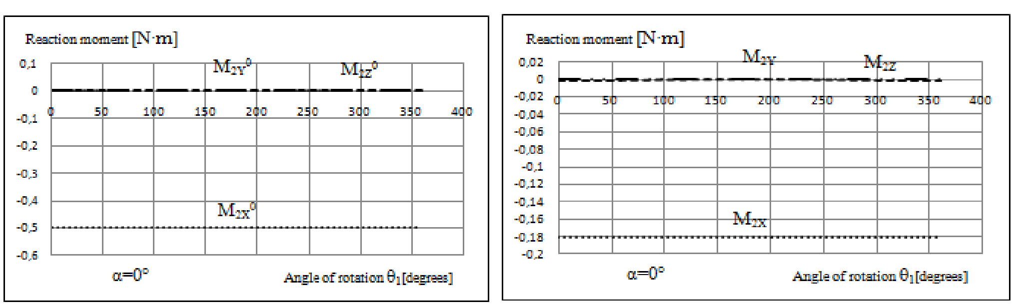


Fig. 7. The variation diagrams of reaction moments in joint 2, for *α=0°* a) in general system of reference b) in local system of reference

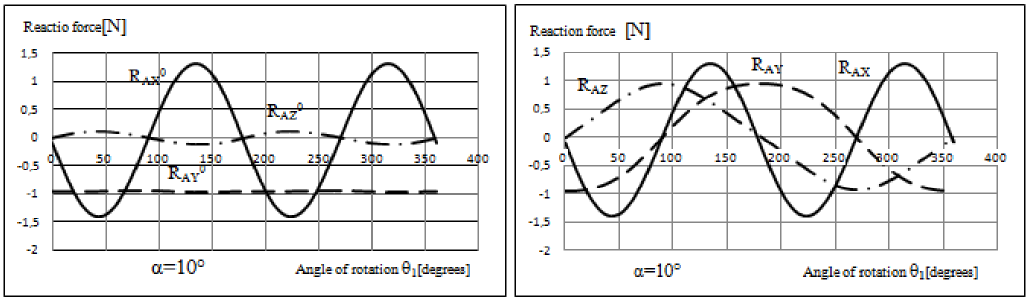


Fig. 8. The variation diagrams of reaction forces in joint A, for *α=10°* a) in general system of reference b) in local system of reference

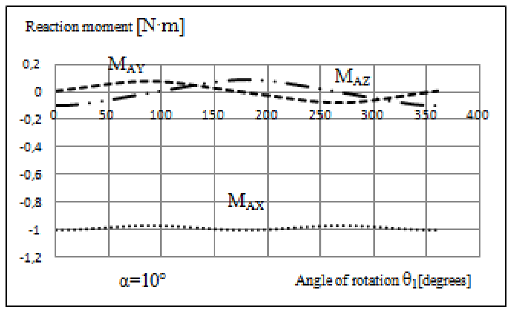


Fig. 9. The variation diagrams of reaction moments in joint A for *α=10°* in local system of reference

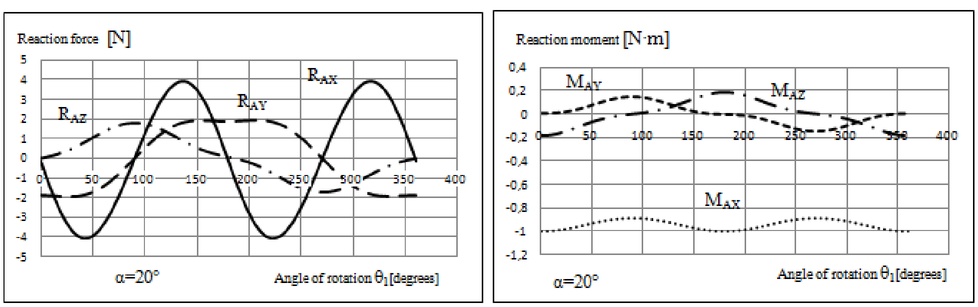


Fig. 10. The variation diagrams of reaction forces and moments in joint A, for *α=20°* in local system of reference

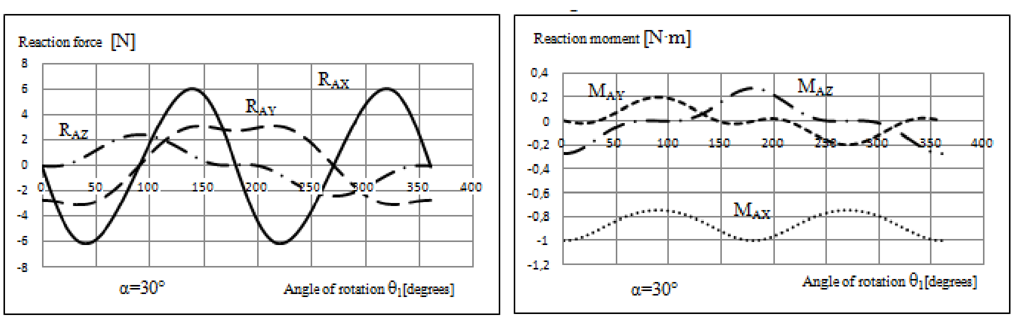


Fig. 11. The variation diagrams of reaction forces and moments in joint A, for *α=30°* in local system of reference

It is observed that the maxim value of the axial reaction R_{AX} from kinematic pairs A, increases from 0N ($\alpha = 0^\circ$) to 1.4N ($\alpha = 10^\circ$), to 4N ($\alpha = 20^\circ$) and to 6N ($\alpha = 30^\circ$).

4. CONCLUSION

The elastic calculation and the relative displacements method makes possible the determination of the reactions from the kinematic pairs of the 4R spherical quadrilateral mechanism, that is multiple statically indeterminate Following the numerical simulations the following conclusions can be taken

1. For α=0°, in the general system of reference $OX_0Y_0Z_0$ the reactions force and moments after axes directions OY_0 and OZ_0 vary with the angle θ_1, and in the local system of reference they remain constant. The axial forces (after axe direction OX_0) from all kinematic pairs in general system of reference have the zero value (they are balanced), in contrast to the 4r asymmetrical spherical quadrilateral mechanism where in all kinematic pairs appears an unbalanced axial force [11,12].
2. For α≠0°, both in local and general reference systems, the reactions and moments aren't constant and have a variation depending on the angle θ_1.
3. The forces and moments from the reaction are increasing as the angle of inclination α increases.

COMPETING INTERESTS

Author has declared that no competing interests exist.

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