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Existence and Structure of Entire Large Positive Radial Solutions for A Class of Quasilinear Elliptic Systems

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Abstract

In this paper, we consider the problem for the existence of the entire positive radial solutions of quasilinear elliptic system

$$\begin{cases} \text{div}(A_1(|\nabla u|)\nabla u) = F(|x|, u, v), x \in \mathbf{R}^N, \\ \text{div}(A_2(|\nabla v|)\nabla v) = G(|x|, u, v), x \in \mathbf{R}^N, \\ \lim\limits_{|x|\to\infty} u(x) = +\infty, \lim\limits_{|x|\to\infty} v(x) = +\infty, x \in \mathbf{R}^N. \end{cases}$$

And, we also consider the following quasilinear elliptic system:

$$\begin{cases} \text{div}(A_1(|\nabla u|)\nabla u) = p(|x|)g(v), x \in \mathbf{R}^N, \\ \text{div}(A_2(|\nabla v|)\nabla v) = q(|x|)f(u), x \in \mathbf{R}^N, \\ \lim\limits_{|x|\to\infty} u(x) = +\infty, \lim\limits_{|x|\to\infty} v(x) = +\infty, x \in \mathbf{R}^N. \end{cases}$$

Using the theory of ordinary differential equation, iteration method and comparison principle, which studied the existence and structure of positive entire large radial solutions. The main results of the present paper are new and extend the previously known results.

Keywords: Quasilinear elliptic systems; Existence and structure; Entire large positive solution.

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1 Introduction

In this paper, we are concerned with the existence of entire positive radial solutions to the following quasilinear elliptic system of the form

$$\left\{\begin{array}{l} \text{div}(A_1(|\nabla u|)\nabla u) = F(|x|, u, v), x \in \mathbf{R}^N, \\ \text{div}(A_2(|\nabla v|)\nabla v) = G(|x|, u, v), x \in \mathbf{R}^N, \\ \lim\limits_{|x|\to\infty} u(x) = +\infty, \lim\limits_{|x|\to\infty} v(x) = +\infty, x \in \mathbf{R}^N. \end{array}\right. \tag{1.1}$$

where $N \geq 3$, $A_1, A_2 \in C^2(0, \infty)$, F, G are nonnegative continuous functions on $\mathbf{R}^N \times [0, \infty) \times [0, \infty)$. And, we also consider the following quasilinear elliptic system:

$$\left\{\begin{array}{l} \text{div}(A_1(|\nabla u|)\nabla u) = p(|x|)g(v), x \in \mathbf{R}^N, \\ \text{div}(A_2(|\nabla v|)\nabla v) = q(|x|)f(u), x \in \mathbf{R}^N, \\ \lim\limits_{|x|\to\infty} u(x) = +\infty, \lim\limits_{|x|\to\infty} v(x) = +\infty, x \in \mathbf{R}^N. \end{array}\right. \tag{1.2}$$

where $N \geq 3$, $A_1, A_2 \in C^2(0, \infty)$, and $p, q \in C(\mathbf{R}^N)$ are positive functions. Throughout this paper we assume that $f, g \in C[0, \infty)$ are positive and non-decreasing on $(0, \infty)$.

When $A_1(|\nabla u|) = |\nabla u|^{p-2}, A_2(|\nabla v|) = |\nabla v|^{q-2}, F(|x|, u, v) = m(|x|)f(u, v)$ and $G(|x|, u, v) = n(|x|)g(u, v)$, where $m, n \in C(\mathbf{R}^N)$ are positive functions. the system (1.1) becomes

$$\left\{\begin{array}{l} \text{div}(|\nabla u|^{p-2}\nabla u) = m(|x|)f(u, v), x \in \mathbf{R}^N, \\ \text{div}(|\nabla v|^{q-2}\nabla v) = n(|x|)g(u, v), x \in \mathbf{R}^N, \end{array}\right. \tag{1.3}$$

Problem (1.3) arises in the theory of quasiregular and quasiconformal mappings as well as in the study of non-Newtonian fluids. In latter case, the pair (p, q) is a characteristic of the medium. Media with $(p, q) > (2, 2)$ are called dilatant fluids and those with $(p, q) < (2, 2)$ are called pseudoplastics.If $(p, q) = (2, 2)$, they are Newtonian fluids. When $(p, q) \neq (2, 2)$, the problem becomes more complicated since certain nice properties inherent to the case $(p, q) = (2, 2)$ seem to be lose or at least difficult to verify. The main differences between $(p, q) = (2, 2)$ and $(p, q) \neq (2, 2)$ can be found in [1], [2], [3], [4], [8], [10], [11], [12].

When $(p, q) = (2, 2)$, the existence or nonexistence of positive solutions for elliptic systems

$$\left\{\begin{array}{l} \Delta u = m(|x|)f(u, v), x \in \mathbf{R}^N, \\ \Delta v = n(|x|)g(u, v), x \in \mathbf{R}^N, \end{array}\right. \tag{1.4}$$

had been studied by [13], [15], [16] and the references therein. Among these literature, Lair and Wood [8] studied the following systems

$$\left\{\begin{array}{l} \Delta u = m(|x|)v^\alpha, x \in \mathbf{R}^N, \\ \Delta v = n(|x|)u^\beta, x \in \mathbf{R}^N, \\ 0 < \alpha \leq 1, 0 < \beta \leq 1. \end{array}\right. \tag{1.5}$$

which using the theory of ordinary differential equations and the comparision principle, proved that all positive entire radial solutions are explosive provided that

$$\int_0^\infty tm(t)dt = \infty, \quad \int_0^\infty tn(t)dt = \infty.$$

On the other hand, if

$$\int_0^\infty tm(t)dt < \infty, \quad \int_0^\infty tn(t)dt < \infty$$

then all positive solutions are bounded. Cristea and Radulescu [13] extended the above results to a large class of systems

$$\begin{cases} \Delta u = m(|x|)g(v), x \in \mathbf{R}^N, \\ \Delta v = n(|x|)f(u), x \in \mathbf{R}^N, \end{cases} \tag{1.6}$$

When $(p, q) \neq (2, 2)$, $A_1(|\nabla u|) = |\nabla u|^{p-2}, A_2(|\nabla v|) = |\nabla v|^{q-2}, F(|x|, u, v) = m(|x|)v^\alpha$ and $G(|x|, u, v) = n(|x|)u^\beta$, the system (1.1) becomes

$$\begin{cases} \text{div}(|\nabla u|^{p-2}\nabla u) = m(|x|)v^\alpha, x \in \mathbf{R}^N, \\ \text{div}(|\nabla v|^{q-2}\nabla v) = n(|x|)u^\beta, x \in \mathbf{R}^N, \end{cases} \tag{1.7}$$

for which existence results for entire explosive positive solution can be found in the paper by YANG Z.D and LU Q.S [3]. Zuodong YANG and Qishao LU established that all entire positive radial solutions of (1.7) are explosive provided that

$$\int_0^\infty \left(t^{1-N} \int_t^\infty s^{N-1} m(s)ds \right)^{\frac{1}{p-1}} dt = \infty$$

$$\int_0^\infty \left(t^{1-N} \int_t^\infty s^{N-1} n(s)ds \right)^{\frac{1}{q-1}} dt = \infty$$

If, on the other hand,

$$\int_0^\infty \left(t^{1-N} \int_t^\infty s^{N-1} m(s)ds \right)^{\frac{1}{p-1}} dt < \infty$$

$$\int_0^\infty \left(t^{1-N} \int_t^\infty s^{N-1} n(s)ds \right)^{\frac{1}{q-1}} dt < \infty$$

then all entire positive radial solutions of (1.7) are bounded.

Zuodong Yang [1] extend the above results to a larger class of systems

$$\begin{cases} \text{div}(|\nabla u|^{p-2}\nabla u) = m(|x|)g(v), x \in \mathbf{R}^N, \\ \text{div}(|\nabla v|^{q-2}\nabla v) = n(|x|)f(u), x \in \mathbf{R}^N, \end{cases} \tag{1.8}$$

Recently, Garcia-Melian J and Rossi J.D [12] studied the existence and nonexistence results for boundary blow-up solutions which can be obtained to the elliptic system

$$\begin{cases} \Delta u = u^{m_1}v^{n_1}, x \in \Omega \subset \mathbf{R}^N, \\ \Delta v = u^{m_2}v^{n_2}, x \in \Omega \subset \mathbf{R}^N, \end{cases} \tag{1.9}$$

Yang Z.D and Wu M.Z [7] extended the above results to the following quasilinear elliptic system

$$\begin{cases} \text{div}(|\nabla u|^{p-2}\nabla u) = u^{m_1}v^{n_1}, x \in \Omega \subset \mathbf{R}^N, \\ \text{div}(|\nabla v|^{q-2}\nabla v) = u^{m_2}v^{n_2}, x \in \Omega \subset \mathbf{R}^N, \end{cases} \tag{1.10}$$

where $N \geq 3, p, q > 1, m_1 > p - 1, n_2 > q - 1, n_1, m_2 > 0$ are real numbers and $\Omega \subset \mathbf{R}^N$ is bounded of class $C^{2,\eta}$ for some $\eta, 0 < \eta < 1$, subject to three different types of Dirichlet boundary conditions: $u = \lambda, v = \mu$ or $u = v = \infty$ or $u = \infty, v = \mu$ on $\partial\Omega$, where $\lambda, \mu > 0$. Under severral hypotheses on the parameters m_1, n_1, m_2, n_2 which is critical case, they show that existence of positive solutions.

Motivated by the results of above cited papers, we further study the existence of positive solutions of (1.1) and (1.2), the results of the (p, q)-Lalacian systems are extended to a large class quasilinear systems. Using an argument inspired by [1], [3], [14], we obtain the following main results which complement and extended to corresponding results in [1], [3].

Throughout this paper, we assume that $A_i (i = 1, 2), F, G, f$ and g satisfy the following hypothesis:

H1 *F, G are nonnegative continuous functions on $\mathbf{R}^N \times [0, \infty) \times [0, \infty)$; $f, g \in C[0, \infty)$ are positive and non-decreasing on $(0, \infty)$.*

H2 *Suppose that the function $A_i : (0, \infty) \to [0, \infty)$ satisfies the regularity requirement $A_i \in C^2(0, \infty)$ and the (possibly degenerate) elliptic condition*

$$\begin{cases} \lim\limits_{t\to 0^+} tA_i(t) = 0, \\ (tA_i(t))' > 0, \forall t > 0. \end{cases} \tag{1.11}$$

From Eq.(1.11) we see that there exists a nonnegative inverse function B_i such that $B_i(\xi) = A_i(\xi)\xi (i = 1, 2)$ on $[0, \infty)$, and denote

$$\begin{aligned} \Phi(\tau, u(\tau), v(\tau)) &:= B_1^{-1}\Big(\tau^{1-N}\int_0^\tau s^{N-1}F(s, u(s), v(s))ds\Big), \\ \Psi(\tau, u(\tau), v(\tau)) &:= B_2^{-1}\Big(\tau^{1-N}\int_0^\tau s^{N-1}G(s, u(s), v(s))ds\Big), \\ \Phi_1(\tau, v(\tau)) &:= B_1^{-1}\Big(\tau^{1-N}\int_0^\tau s^{N-1}p(s)g(v(s))ds\Big), \\ \Psi_1(\tau, u(\tau)) &:= B_2^{-1}\Big(\tau^{1-N}\int_0^\tau s^{N-1}q(s)f(u(s))ds\Big), \\ \phi(\tau, u(\tau)) &:= B_1^{-1}\Big(\tau^{1-N}\int_0^\tau s^{N-1}p(s)f(u(s))ds\Big), \end{aligned}$$

where $\Phi(\tau, u, v)$, $\Psi(\tau, u, v)$, $F(\tau, u, v)$ and $G(\tau, u, v)$ are positive continuous functions on $[0, \infty) \times [0, \infty) \times [0, \infty)$ and are increasing for u and v, $\Phi_1(\tau, v)$, $\Psi_1(\tau, u,)$, $\phi(\tau, u)$, are positive continuous functions on $[0, \infty) \times [0, \infty)$ and are increasing for u and v.

H3 *Suppose that there exist $m, m_1, n, n_1, u, v \in C[0, \infty), u(r), v(r) > 0, \mu, \mu_1 > 0$ such that:*

$$\begin{cases} m(r)u(r) \le \Phi \le m(r)u(r) + n(r)v(r)e^{\mu r}, \\ n(r)v(r) \le \Psi \le m(r)u(r)e^{-\mu r} + n(r)v(r), \end{cases}$$

and

$$\begin{cases} 0 \le \Phi_1 \le m_1(r)u(r) + n_1(r)v(r)e^{\mu_1 r}, \\ 0 \le \Psi_1 \le m_1(r)u(r)e^{-\mu_1 r} + n_1(r)v(r). \end{cases}$$

H4 *Suppose that f and g satisfy the stronger regularity $f, g \in C^1[0, \infty)$,*

$$f(0) = g(0) = 0, \quad \liminf_{u\to\infty} \frac{f(u)}{g(u)} > 0,$$

H5 *Suppose that there exist $m_2, n_2, u, v \in C[0, \infty), u(r), v(r) > 0, \mu_2 > 0$ such that:*

$$0 \le \phi \le m_2(r)u(r) + n_2(r)v(r)e^{\mu_2 r},$$

The purpose of this paper is to extend the results in [1], [3] to a larger class of systems. Using the theory of ordinary differential equation, iteration method and comparison principle. Our main result is summarized in the following theorems.

Theorem 1. *Let **H1** − **H3** be satisfied, then the system (1.1) has infinitely many entire positive solutions* $(u, v) \in C^1([0, \infty))$. *If, in addition, the functions* m *and* n *satisfy*

$$\int_0^\infty m(s)e^{-\mu s}ds = \infty, \quad \int_0^\infty n(s)e^{\mu s}ds = \infty, \tag{1.12}$$

then all entire positive solutions of the system (1.1) are large. On the other hand, if m *and* n *satisfy*

$$\int_0^\infty m(s)ds < \infty, \quad \int_0^\infty n(s)ds < \infty, \tag{1.13}$$

then all entire positive solutions of the system (1.1) are bounded.

Theorem 2. *Let **H1** − **H3** satisfied, then there exists an entire positive radial solution of (1.2) with any central values*

$$u(0) = b > 0, \quad v(0) = c > 0.$$

If, in addition, the functions p *and* q *satisfy*

$$\int_0^\infty B_1^{-1}\Big(t^{1-N}\int_0^t s^{N-1}p(s)g(c)ds\Big)dt = \infty,$$
$$\int_0^\infty B_2^{-1}\Big(t^{1-N}\int_0^t s^{N-1}q(s)f(b)ds\Big)dt = \infty. \tag{1.14}$$

then all entire positive solutions of the system (1.2) are large. On the other hand, if m_1 *and* n_1 *satisfy*

$$\int_0^\infty m_1(s)ds < \infty, \quad \int_0^\infty n_1(s)ds < \infty, \tag{1.15}$$

then all entire positive solutions of the system (1.2) are bounded.

We use the natation $\mathbf{R}^+ = [0, +\infty)$, and define the

$$\mathcal{G} = \{(b, c) \in \mathbf{R}^+ \times \mathbf{R}^+ | u(0) = b, v(0) = c, \ and \ (u, v) \ is \ an \ entire$$
$$radial \ solution \ of \ (1.2)\}$$

Theorem 3. *Let* $f, g \in C^1[0, \infty)$ *satisfy* **H4**. *Assume (1.15) holds,* $\eta(|x|) = \min\{p(|x|), q(|x|)\} \geq C > 0$, *Then set* $\mathcal{G} \neq \emptyset$ *and is a closed bounded subset of* $\mathbf{R}^+ \times \mathbf{R}^+$.

The paper is organized as follows. In Section 2, we collect some Preliminary results which are essential for the proof of the theorems. Section 3 is devoted to the proof of the main results of the present paper.

2 Preliminaries

In this section, we will use Gronwall-Bellman's inequality to deduce an inequality system, that is lemma 2.2 which is a key in the proof of our main results.

Lemma 2.1.(Gronwall-Bellman's inequality, see[5], [6]) Let $u(t)$ and $b(t)$ be nonnegative continuous functions on $J = [0, a], a > 0$, for which the inequality

$$u(t) \leq c + \int_0^t b(s)u(s)ds, t \in J,$$

holds, where $c \geq 0$ is a constant. then

$$u(t) \leq ce^{\int_0^t b(s)ds}, \forall t \in J,$$

Lemma 2.2. Let $u(t), v(t), h_i(t)(i = 1, 2, 3, 4)$ be nonnegative continuous functions on $[0, \infty)$, and let $\theta(t) = \sup_i h_i(t)(i = 1, 2, 3, 4)$ for any $t > 0$. Suppose for any $t \geq 0$,

$$u(t) \leq c_1 + \int_0^t h_1(s)u(s)ds + \int_0^t h_2(s)v(s)e^{\mu s}ds, \tag{2.1}$$

$$v(t) \leq c_2 + \int_0^t h_3(s)u(s)e^{-\mu s}ds + \int_0^t h_4(s)v(s)ds, \tag{2.2}$$

where c_1, c_2, μ are nonnegative constant. Then there exist constant $M > 0$ and nonnegative continuous functions $\rho_k(t)(k = 1, 2)$ on $[0, \infty)$, such that

$$u(t) \leq Me^{\int_0^t \rho_1(s)ds}, v(t) \leq Me^{\int_0^t \rho_2(s)ds}.$$

Proof. First of all, let us make the transformation

$$\widetilde{u}(t) = u(t)e^{\alpha t}, \widetilde{v}(t) = v(t)e^{\beta t}, t \in [0, \infty), \tag{2.3}$$

namely,

$$u(t) = \widetilde{u}(t)e^{-\alpha t}, v(t) = \widetilde{v}(t)e^{-\beta t}, t \in [0, \infty), \tag{2.4}$$

where α, β satisfy $\alpha + \mu = \beta, \alpha > 0$. Thus we substitute (2.4) into (2.1) and (2.3), we obtain

$$\widetilde{u}(t)e^{-\alpha t} \leq c_1 + \int_0^t h_1(s)\widetilde{u}(s)e^{-\alpha s}ds + \int_0^t h_2(s)\widetilde{v}(s)e^{-\beta s}e^{\mu s}ds,$$

$$\widetilde{v}(t)e^{-\beta t} \leq c_2 + \int_0^t h_3(s)\widetilde{u}(s)e^{-\alpha s}e^{-\mu s}ds + \int_0^t h_4(s)\widetilde{v}(s)e^{-\beta s}ds.$$

Since α, β satisfy $\alpha + \mu = \beta$, we have

$$\widetilde{u}(t)e^{-\alpha t} \leq c_1 + \int_0^t h_1(s)\widetilde{u}(s)e^{-\alpha s}ds + \int_0^t h_2(s)\widetilde{v}(s)e^{-\alpha s}ds,$$

$$\widetilde{v}(t)e^{-\beta t} \leq c_2 + \int_0^t h_3(s)\widetilde{u}(s)e^{-\beta s}ds + \int_0^t h_4(s)\widetilde{v}(s)e^{-\beta s}ds.$$

Meanwhile, since $|h_i(t)| \leq \theta(t)(i = 1, 2, 3, 4)$ on $[0, \infty)$, we get

$$\widetilde{u}(t) \leq c_1e^{\alpha t} + \int_0^t \theta(s)e^{\alpha(t-s)}(\widetilde{u}(s) + \widetilde{v}(s))ds,$$

$$\widetilde{v}(t) \leq c_2e^{\beta t} + \int_0^t \theta(s)e^{\beta(t-s)}(\widetilde{u}(s) + \widetilde{v}(s))ds,$$

Thus

$$\begin{aligned} \widetilde{u}(t) + \widetilde{v}(t) &\leq c_1e^{\alpha t} + c_2e^{\beta t} + \int_0^t \theta(s)(\widetilde{u}(s) + \widetilde{v}(s))(e^{\alpha(t-s)} + e^{\beta(t-s)})ds \\ &\leq (c_1 + c_2)e^{\beta t} + 2\int_0^t \theta(s)(\widetilde{u}(s) + \widetilde{v}(s))e^{\beta(t-s)}ds, \end{aligned}$$

Namely,

$$(\widetilde{u}(t)+\widetilde{v}(t))e^{-\beta t} \leq c_1 + c_2 + 2\int_0^t \theta(s)(\widetilde{u}(s)+\widetilde{v}(s))e^{-\beta s}ds.$$

By Gronwall-Bellman's inequality, we get

$$(\widetilde{u}(t)+\widetilde{v}(t))e^{-\beta t} \leq Me^{2\int_0^t \theta(s)ds}, \tag{2.5}$$

where $M = c_1 + c_2$. Thus, from (2.4) and (2.5), we get

$$\begin{aligned} v(t) &= \widetilde{v}(t)e^{-\beta t} \leq Me^{2\int_0^t \theta(s)ds}, \\ u(t) &= \widetilde{u}(t)e^{-\alpha t} = \widetilde{u}(t)e^{-\beta t + \mu t} \\ &\leq Me^{2\int_0^t (\theta(s)+\frac{1}{2}\mu)ds}. \end{aligned}$$

Thus the proof of lemma 2.2 is completed now.

Lemma 2.3 (Weak comparison principle, see [13]). Let Ω be a bounded domain in $\mathbf{R}^N (N \geq 2)$ with smooth boundary $\partial\Omega$ and $\theta : (0,\infty) \to (0,\infty)$ is continuous and non-decreasing. Let $u_1, u_2 \in W^{1,m}(\Omega)$ satisfy

$$\int_\Omega A_1(|\nabla u_1|)\nabla u_1 \nabla\psi dx + \int_\Omega \theta(u_1)\psi dx \leq \int_\Omega A_1(|\nabla u_2|)\nabla u_2 \nabla\psi dx + \int_\Omega \theta(u_2)\psi dx,$$

for all non-negative $\psi \in W_0^{1,m}(\Omega)$. Then the inequality

$$u_1 \leq u_2 \quad on \ \partial\Omega$$

implies that

$$u_1 \leq u_2 \quad in \ \Omega.$$

Lemma 2.4 Let **H5** be satisfied, $f \in C^1[0,\infty),\ f(0) = 0,\ f'(u) \geq 0$ and $p(|x|) \geq C > 0$ for $x \in \mathbf{R}^N$ and the following:

$$\int_0^\infty B_1^{-1}\left(f(a)t^{1-N}\int_0^t s^{N-1}p(s)ds\right)dt = \infty, \ \text{ for all } \forall a > 0. \tag{2.6}$$

Then equation

$$\text{div}(A_1(|\nabla u|)\nabla u) = p(|x|)f(u) \tag{2.7}$$

has an entire large positive radial solution.

Proof. *Step1.* We start by showing that (2.7) has an positive radial solutions. To do this, we shall first fix $u(0) = a$ and then show that the following ordinary differential equation system

$$\begin{cases} (A_1(|u'|)u')' + \frac{N-1}{r}(A_1(|u'|)u') = p(|x|)f(u(r)), r \geq 0, \\ u'(r) \geq 0, \text{ on } [0,\infty), \\ u(0) = a > 0 \end{cases} \tag{2.8}$$

has a solutions u. Thus $u(x) = u(|x|)$ are positive solutions of equation (16). Integrating (2.8), we have

$$\begin{aligned} u(r) &= a + \int_0^r B_1^{-1}\left(t^{1-N}\int_0^t s^{N-1}p(s)f(u(s))\right)dt \\ &= a + \int_0^r \phi(s,u(s))ds, \forall r \geq 0, \end{aligned} \tag{2.9}$$

Let $\{u_k\}_{k\geq 0}$ be sequences of positive continuous functions defined on $[0,\infty)$ by

$$\begin{aligned} u_0 &= a, \\ u_k(r) &= a + \int_0^r \phi(s, u_{k-1}(s))ds, \forall r \geq 0, \end{aligned} \tag{2.10}$$

Since $u_1(r) \geq a$ and $\phi(\tau, u)$ is increasing for u, we find $u_2(r) \geq u_1(r)$ for all $r \geq 0$. Proceeding at the same manner we conclude that

$$u_k(r) \leq u_{k+1}(r), \forall r > 0, k \geq 1,$$

and hence the sequences $\{u_k\}_{k\geq 0}$ is increasing on $[0,\infty)$.

Since **H5** be satisfied, we have

$$\phi \leq m_2(r)u_{k-1}(r) + n_2(r)v_{k-1}(r)e^{\mu_2 r} \tag{2.11}$$

Consequently, under the hypotheses **H2** and $m_2, n_2 \in C[0,\infty)$, we notice that, for each $r > 0$,

$$u_k(r) \leq a + \int_0^r m_2(s)u_k(s)ds + \int_0^r n_2(s)v_k(s)e^{\mu_2 s}ds,$$

Thus, by Lemma 2.2, we have

$$u_k(r) \leq Me^{\int_0^r \beta_1(s)ds}. \tag{2.12}$$

where $M > 0, \beta_1(t) = 2\max\{|m_2(t)|, |n_2(t)|\}$

Consequently, by (2.12), the sequences u_k is uniformly bounded and increasing on $[0, R)$ for any $R > 0$. By Arzela-Ascoli theorem, we obtain easily that the sequences u_k is converge uniformly to u on $[0, R)$ for any $R > 0$. Consequently, u is a positive solution of (2.8), that is , u is an entire positive radial solution of (2.7).

Step2. From (2.9), we get

$$u(r) \quad \geq \quad a + \int_0^r B_1^{-1}\left(f(a)t^{1-N}\int_0^t s^{N-1}p(s)ds\right)dt \tag{2.13}$$

Thus, by (2.6) and (2.13), we have

$$\begin{aligned} \lim_{r\to\infty} u(r) &\geq a + \int_0^\infty B_1^{-1}\left(f(a)t^{1-N}\int_0^t s^{N-1}p(s)ds\right)dt \\ &= \infty, \end{aligned} \tag{2.14}$$

We get that the entire positive solutions of the equation (2.7) is large.

Corollary 2.5 Let **H5** be satisfied, $f \in C^1[0,\infty)$, $f(0) = 0$, $f'(u) \geq 0$ and $p(|x|) \geq C > 0$ for $x \in B(0, R)$,$B(0, R)$ is a bounded domain, and the following:

$$\int_0^R B_1^{-1}\left(f(a)t^{1-N}\int_0^t s^{N-1}p(s)ds\right)dt = \infty,$$

Then equation

$$\text{div}(A_1(|\nabla u|)\nabla u) = p(|x|)f(u)$$

has an entire large positive radial solution.

Proof. The proof is similar to the Lemma2.4.

Corollary 2.6 The problem

$$\text{div}(A_1(|\nabla l|)\nabla l) = (p(|x|)+q(|x|))(f(l)+g(l)), \quad (2.15)$$

$$\text{div}(A_2(|\nabla h|)\nabla h) = (p(|x|)+q(|x|))(f(h)+g(h)), \quad (2.16)$$

has an entire large positive radial solution provided that function $f \in C^1[0,\infty)$, $f(0)=0$, $f'(u) \geq 0$ and $\eta(|x|) \geq C > 0$ for $x \in B(0,R)$,$B(0,R)$ is a bounded domain, and p, q satisfy **H5**.

Lemma 2.7 Let l, h be any entire positive radial solution of (2.15), (2.16) given in Corollary 2.6 and define the sequences $\{u_k\}$ and $\{v_k\}$ by

$$\begin{aligned} u_k(r) &= b + \int_0^r B_1^{-1}\Big(t^{1-N}\int_0^t s^{N-1}p(s)g(v_{k-1}(s))ds\Big)dt, \\ &= b + \int_0^r \Phi_1(s, v_{k-1}(s))ds, \quad r \geq 0, \quad (2.17) \\ v_k(r) &= c + \int_0^r B_2^{-1}\Big(t^{1-N}\int_0^t s^{N-1}q(s)f(u_{k-1}(s))ds\Big)dt, \\ &= c + \int_0^r \Psi_1(s, u_{k-1}(s))ds, \quad r \geq 0, \quad (2.18) \end{aligned}$$

where $u_0 = b, 0 \leq b \leq \min\{l(0), h(0)\}$ and $v_0 = c, 0 \leq c \leq \min\{l(0), h(0)\}$. Then

(a)$u_k(r) \leq u_{k+1}(r)$ and $v_k(r) \leq v_{k+1}(r), r \in \mathbf{R}^+, k \geq 1,$ and
(b)$u_k(r) \leq l(r)$ and $v_k(r) \leq h(r), r \in \mathbf{R}^+, k \geq 1.$

Thus $\{u_k(r)\}$ and $\{v_k(r)\}$ converge and the limit functions are positive radial solutions of system (1.2).

Proof. (a) Obviously, $v_0 < v_1$. This then yields $u_1 < u_2$ by (28) and **H2**. Consequently, $v_1 < v_2$ by (29) and **H2**, which yields $u_2 < u_3$ by (28). Continuing this line of reasoning, we obtain that $\{u_k\}$ and $\{v_k\}$ are monotonically increasing.
(b) We note first that since l, h is radial, we get

$$l(r) = l(0) + \int_0^r B_1^{-1}\Big(t^{1-N}\int_0^t s^{N-1}(p(s)+q(s))(f(l(s))+g(l(s)))ds\Big)dt,$$

From **H2**, we get that

$$l'(r) = B_1^{-1}\Big(r^{1-N}\int_0^r s^{N-1}(p(s)+q(s))(f(l(s))+g(l(s)))ds\Big) > 0,$$

Now, it is clear that $c = v_0 \leq l(0) \leq l(r)$ for all $r \geq 0$. Thus, we have

$$\begin{aligned} u_1(r) &= b + \int_0^r B_1^{-1}\Big(t^{1-N}\int_0^t s^{N-1}p(s)g(v_0))ds\Big)dt \\ &\leq l(0) + \int_0^r B_1^{-1}\Big(t^{1-N}\int_0^t s^{N-1}(p(s)+q(s))(f(l(s))+g(l(s)))ds\Big)dt \\ &= l(r). \end{aligned}$$

Thus we have $u_1 \leq l(r)$. Similar arguments will show, in sequence, that $v_1 \leq h(r),\ u_2 \leq l(r),\ \cdots.$

3 Proof of main Theorems

In this section, we will give the proof of the main results.

Proof of Theorem 1. *Step1.* We start by showing that (1) has positive radial solutions. To do this, we shall first fix $b > a$ and $c > a$ and then show that the following ordinary differential equation system

$$\begin{cases} (A_1(|u'|)u')' + \frac{N-1}{r}(A_1(|u'|)u') = f(r, u(r), v(r)), r \geq 0, \\ (A_2(|v'|)v')' + \frac{N-1}{r}(A_2(|v'|)v') = g(r, u(r), v(r)), r \geq 0, \\ u'(r) \geq 0, v'(r) \geq 0, on\ [0, \infty), \\ u(0) = b > 0, v(0) = c > 0. \end{cases} \tag{3.1}$$

has solutions (u, v). Thus $u(x) = u(|x|), v(x) = v(|x|)$ are positive solutions of system (1). Integrating (3.1), we have

$$\begin{aligned} u(r) &= b + \int_0^r B_1^{-1}\Big(t^{1-N}\int_0^t s^{N-1}F(s, u(s), v(s))\Big)dt \\ &= b + \int_0^r \Phi(s, u(s), v(s))ds, \forall r \geq 0, \end{aligned} \tag{3.2}$$

$$\begin{aligned} v(r) &= c + \int_0^r B_2^{-1}\Big(t^{1-N}\int_0^t s^{N-1}G(s, u(s), v(s))\Big)dt \\ &= c + \int_0^r \Psi(s, u(s), v(s))ds, \forall r \geq 0, \end{aligned} \tag{3.3}$$

Let $\{u_k\}_{k\geq 0}$ and $\{v_k\}_{k\geq 0}$ be sequences of positive continuous functions defined on $[0, \infty)$ by

$$u_0 \equiv b,\ v_0 \equiv c,$$

$$u_k(r) = b + \int_0^r \Phi(s, u_{k-1}(s), v_{k-1}(s))ds, \forall r \geq 0, \tag{3.4}$$

$$v_k(r) = c + \int_0^r \Psi(s, u_{k-1}(s), v_{k-1}(s))ds, \forall r \geq 0. \tag{3.5}$$

Since **H3** holds, we have

$$m(r)u_{k-1}(r) \leq \Phi(s, u_{k-1}(s), v_{k-1}(s)) \leq m(r)u_{k-1}(r) + n(r)v_{k-1}(r)e^{\mu r}, \tag{3.6}$$

$$n(r)v_{k-1}(r) \leq \Psi(s, u_{k-1}(s), v_{k-1}(s)) \leq m(r)u_{k-1}(r)e^{-\mu r} + n(r)v_{k-1}(r). \tag{3.7}$$

Consequently,

$$b + \int_0^r m(s)u_{k-1}(s)ds \leq u_k(r) \leq b + \int_0^r m(s)u_{k-1}(s)ds + \int_0^r n(s)v_{k-1}(s)e^{\mu s}ds, \tag{3.8}$$

$$c + \int_0^r n(s)v_{k-1}(s)ds \leq v_k(r) \leq c + \int_0^r m(s)u_{k-1}(s)e^{-\mu s}ds + \int_0^r n(s)v_{k-1}(s)ds. \tag{3.9}$$

Under the hypotheses **H1, H2** and $m, n \in C[0, \infty)$, it is easy to see that $u_1(r) \geq u_0,\ v_1(r) \geq v_0, \forall r > 0$. We substitute these into (3.4) and (3.5) , and use the hypotheses **H2** again to obtain $u_2(r) \geq u_1(r), v_2(r) \geq v_1(r), \forall r > 0$.

By the same argument, we deduce that

$$u_k(r) \leq u_{k+1}(r), v_k(r) \leq v_{k+1}(r), \forall r > 0, k \geq 1,$$

and hence the sequences $\{u_k\}_{k\geq 0}$ and $\{v_k\}_{k\geq 0}$ are increasing on $[0,\infty)$. Moreover, under the hypotheses **H1, H2** and $m,n \in C[0,\infty)$, we notice that, for each $r>0$,

$$u_k(r) \leq b + \int_0^r m(s)u_k(s)ds + \int_0^r n(s)v_k(s)e^{\mu s}ds,$$

and

$$v_k(r) \leq c + \int_0^r m(s)u_k(s)e^{-\mu s}ds + \int_0^r n(s)v_k(s)ds,$$

Thus, by lemma2.2, we have

$$u_k(r) \leq Me^{\int_0^r \beta_1(s)ds}, v_k(r) \leq Me^{\int_0^r \beta_2(s)ds}, \tag{3.10}$$

where $M>0, \beta_1(t) = 2\max\{|m(t)|,|n(t)|\}, \beta_2(t) = 2\max\{|m(t)|+\frac{1}{2}\mu, |n(t)|+\frac{1}{2}\mu\}$.

Consequently, by (3.10), the sequences u_k and v_k are uniformly bounded and increasing on $[0,R)$ for any $R>0$. By Arzela-Ascoli theorem, we obtain easily that the sequences u_k and v_k are converge uniformly to u and v on $[0,R)$ for any $R>0$. Consequently, (u,v) is a positive solution of (3.1), that is , (u,v) is an entire positive radial solution of (1.1). Notice $u(0)=b, v(0)=c$, and $(b,c)\in(0,\infty)\times(0,\infty)$ was chosen arbitrarily, it follows that (1.1) has infinitely many positive entire solutions.

Step2. From (3.6), we get

$$\begin{aligned} b + \int_0^r m(s)u_{k-1}(s)e^{-\mu s}ds &\leq b + \int_0^r \Phi e^{-\mu s}ds \\ &\leq b + \int_0^r \Phi ds \\ &= u(r) \end{aligned} \tag{3.11}$$

Thus, by (1.12) and (3.11), we have

$$\begin{aligned} u(\infty) &\geq \lim_{k\to\infty}(b + \int_0^\infty m(s)u_{k-1}(s)e^{-\mu s}ds) \\ &= b + b\int_0^\infty m(s)e^{-\mu s}ds \\ &= \infty, \end{aligned} \tag{3.12}$$

A similar argument shows that:

$$\lim_{r\to\infty} v(r) = \infty.$$

We get that all entire positive solutions of the system (1.1) are large.

Step3. Let us now drop the condition (1.12) and assume that (1.13) is fulfilled. Using (1.13) and (3.10) we obtain

$$u(r) = \lim_{k\to\infty} u_k(r) \leq Me^{\int_0^r \beta_1(s)ds},$$
$$v(r) = \lim_{k\to\infty} v_k(r) \leq Me^{\int_0^r \beta_2(s)ds}.$$

Taking $r\to\infty$ we get that all entire positive solutions of system (1.1) are bounded. This concludes the proof of Theorem1.

Proof of Theorem 2. *Step1.* Since the radial solutions of (1.2) are solutions of the ordinary differential equations system

$$\begin{array}{l}(A_1(|u'|)u')' + \frac{N-1}{r}(A_1(|u'|)u') = p(r)g(v(r)), r \geq 0,\\ (A_2(|v'|)v')' + \frac{N-1}{r}(A_2(|v'|)v') = q(r)f(u(r)), r \geq 0,\end{array} \tag{3.13}$$

it follows that the radial solutions of (1.2) with $u(0) = b > 0, v(0) = c > 0$ satisfy

$$\begin{aligned} u(r) &= b + \int_0^r B_1^{-1}\Big(t^{1-N}\int_0^t s^{N-1}p(s)g(v(s))\Big)dt \\ &= b + \int_0^r \Phi_1(s, v(s))ds, \forall r \geq 0, \end{aligned} \tag{3.14}$$

$$\begin{aligned} v(r) &= c + \int_0^r B_2^{-1}\Big(t^{1-N}\int_0^t s^{N-1}q(s)f(u(s))\Big)dt \\ &= c + \int_0^r \Psi_1(s, u(s))ds, \forall r \geq 0, \end{aligned} \tag{3.15}$$

Define $v_0(r) = c$ for all $r \geq 0$. Let $\{u_k\}_{k\geq 0}$ and $\{v_k\}_{k\geq 0}$ be sequences of positive continuous functions defined on $[0, \infty)$ given by

$$u_k(r) = b + \int_0^r \Phi_1(s, v_{k-1}(s))ds, \forall r \geq 0, \tag{3.16}$$

$$v_k(r) = c + \int_0^r \Psi_1(s, u_{k-1}(s))ds, \forall r \geq 0. \tag{3.17}$$

Since $v_1(r) \geq c$ and the monotonicity of $\Phi_1(t, v)$ and $\Psi_1(t, u)$, we find $u_2(r) \geq u_1(r)$ for all $r \geq 0$. This implies $v_2(r) \geq v_1(r)$ which further produces $u_3(r) \geq u_2(r)$ for all $r \geq 0$. Proceeding at the same manner we conclude that $u_k(r) \leq u_{k+1}(r)$ and $v_k(r) \leq v_{k+1}(r), \forall r \geq 0$ and $k \geq 1$.

Since **H3** holds, we have

$$\Phi_1(s, v_{k-1}(s)) \leq m_1(r)u_{k-1}(r) + n_1(r)v_{k-1}(r)e^{\mu_1 r}, \tag{3.18}$$

$$\Psi_1(s, u_{k-1}(s)) \leq m_1(r)u_{k-1}(r)e^{-\mu_1 r} + n_1(r)v_{k-1}(r). \tag{3.19}$$

Consequently,

$$u_k(r) \leq b + \int_0^r m_1(s)u_{k-1}(s)ds + \int_0^r n_1(s)v_{k-1}(s)e^{\mu_1 s}ds, \tag{3.20}$$

$$v_k(r) \leq c + \int_0^r m_1(s)u_{k-1}(s)e^{-\mu_1 s}ds + \int_0^r n_1(s)v_{k-1}(s)ds. \tag{3.21}$$

Moreover, under the hypotheses **H1, H2** and $m_1, n_1 \in C[0, \infty)$, we notice that, for each $r > 0$,

$$u_k(r) \leq b + \int_0^r m_1(s)u_k(s)ds + \int_0^r n_1(s)v_k(s)e^{\mu_1 s}ds,$$

and

$$v_k(r) \leq c + \int_0^r m_1(s)u_k(s)e^{-\mu_1 s}ds + \int_0^r n_1(s)v_k(s)ds,$$

Thus, by lemma2.2, we have

$$u_k(r) \leq Me^{\int_0^r \beta_1(s)ds}, v_k(r) \leq Me^{\int_0^r \beta_2(s)ds}, \tag{3.22}$$

where $M > 0, \beta_1(t) = 2\max\{|m_1(t)|, |n_1(t)|\}, \beta_2(t) = 2\max\{|m_1(t)| + \frac{1}{2}\mu_1, |n_1(t)| + \frac{1}{2}\mu_1\}$.

Consequently, by (3.22), the sequences u_k and v_k are uniformly bounded and increasing on $[0, R)$ for any $R > 0$. By Arzela-Ascoli theorem, we obtain easily that the sequences u_k and v_k are converge uniformly to u and v on $[0, R)$ for any $R > 0$. Consequently, (u, v) is a positive solution of (3.13), that is , (u, v) is an entire positive radial solution of (1.2).

Step2. Let (u, v) be an entire positive radial solution of (1.2). Using (3.14) and (3.15) we obtain

$$u(r) \geq b + \int_0^r B_1^{-1}\Big(t^{1-N}\int_0^t s^{N-1}p(s)g(c)ds\Big)dt,$$

$$v(r) \geq c + \int_0^r B_2^{-1}\Big(t^{1-N}\int_0^t s^{N-1}q(s)f(b)ds\Big)dt.$$

Taking $r \to \infty$ we get that (u, v) is an entire large solution.

Step3. Let us now drop the condition (1.14) and assume that (1.15) is fulfilled. Using (1.15) and (3.22) we obtain

$$u(r) = \lim_{k\to\infty} u_k(r) \leq Me^{\int_0^r \beta_1(s)ds},$$

$$v(r) = \lim_{k\to\infty} v_k(r) \leq Me^{\int_0^r \beta_2(s)ds}.$$

Taking $r \to \infty$ we get that all entire positive solutions of system (1.2) are bounded. This concludes the proof of Theorem 2.

Proof of Theorem 3. From Lemma 2.7, it is clear that $[0, l(0)] \times [0, h(0)] \subset \mathcal{G}$ so that $\mathcal{G}$ is non-empty. We shall show that $\mathcal{G}$ is a bounded, closed set.

As a preliminary, note that if $(b, c) \in \mathcal{G}$ then any pair (b_0, c_0) for which $0 \leq b_0 \leq b$ and $0 \leq c_0 \leq c$ must be in $\mathcal{G}$ since the process used in Lemma 7 can be repeated with

$$\begin{aligned} u_k(r) &= b_0 + \int_0^r B_1^{-1}\Big(t^{1-N}\int_0^t s^{N-1}p(s)g(v_{k-1}(s))ds\Big)dt, \\ &= b_0 + \int_0^r \Phi_1(s, v_{k-1}(s))ds, \end{aligned} \tag{3.23}$$

$$\begin{aligned} v_k(r) &= c_0 + \int_0^r B_2^{-1}\Big(t^{1-N}\int_0^t s^{N-1}p(s)f(u_{k-1}(s))ds\Big)dt, \\ &= c_0 + \int_0^r \Psi_1(s, u_{k-1}(s))ds, \end{aligned} \tag{3.24}$$

and $v_0 = c, u_0 = b$. Then,as in Lemma 2.7, the sequences $\{u_k\}$ and $\{v_k\}$ are monotonically increasing. Then, letting (U, V) be the solution of system (2) with central values (b, c), we can easily prove, since $c_0 \leq c$, that $v_0 \leq V$. Thus, $u_1 \leq U$ (since, also, $a_0 \leq a$), and consequently $v_1 \leq V$, and so on. Hence we get $u_k \leq U$ and $v_k \leq V$, and therefore $u \leq U$ and $v \leq V$ where $(u, v) = \lim_{k\to\infty}(u_k, v_k)$ is a solution of (1.2)(with central values (b_0, c_0).

Set $0 < \lambda < 1$ and let $\delta = \delta(\lambda)$ be large enough so that

$$f(t) \geq \lambda g(t), \ \ \forall t \geq \delta. \tag{3.25}$$

Corollary 2.5 ensures the existence of a positive explosive solution h_1, h_2 of the problem

$$\text{div}(A_1(|\nabla h_1|)\nabla h_1) = \lambda\eta(|x|)g(h_1) \ \text{ in } B(0, R),$$

$$h_1 \to \infty, \quad |x| \to R,$$

$$\text{div}(A_2(|\nabla h_2|)\nabla h_2) = \lambda\eta(|x|)g(h_2) \ \text{ in } B(0, R),$$

$$h_2 \to \infty, \quad |x| \to R,$$

To prove that $\mathcal{G}$ is bounded, assume that it is not. Then, there exist $(b, c) \in \mathcal{G}$ such that $b + c > \max\{2\delta, h_1(0) + h_2(0)\}$. Let (u, v) be the entire radial solution of (1.2) such that $(u(0), v(0)) = (b, c)$. Since $u(x) + v(x) \geq b + c > 2\delta$ for all $x \in \mathbf{R}^N$, by (3.25) we get

$$\text{div}(A_1(|\nabla u|)\nabla u) = p(|x|)g(v) \geq \lambda\eta(|x|)g(v),$$
$$\text{div}(A_2(|\nabla v|)\nabla v) = q(|x|)f(u) \geq \lambda\eta(|x|)f(u).$$

On the other hand, $h_1(x) \to \infty, h_2(x) \to \infty$ as $|x| \to R$. Thus, using Lemma 2.3 we conclude that $u + v \leq h_1 + h_2$ in $B(0, R)$. But this is impossible since $u(0) + v(0) = b + c > h_1(0) + h_2(0)$.

To prove that $\mathcal{G}$ is closed, we let $(b_0, c_0) \in \partial\mathcal{G}$ and show that $(b_0, c_0) \in \mathcal{G}$. Let (u, v) be the solution of system (1.2) which corresponds to $b = b_0$ and $c = c_0$. Without loss of generality, we may assume that $\max\{b_0, c_0\} > C = l(0)$ where the function l is given in Lemma 2.7. If $\max\{b_0, c_0\} = b_0$, then $C \leq b_0 - \frac{1}{k}$ for large k so that $u_k(r) \geq C$ for all $r \geq 0$ and for all k sufficiently large where

$$u_k(r) = a_0 - \frac{1}{k} + \int_0^r B_1^{-1}\left(t^{1-N}\int_0^t s^{N-1}p(s)g(v_{k-1}(s))ds\right)dt,$$
$$v_k(r) = c_0 + \int_0^r B_2^{-1}\left(t^{1-N}\int_0^t s^{N-1}p(s)f(u_{k-1}(s))ds\right)dt,$$

From (3.25), we have

$$\text{div}(A_1(|\nabla u_k|)\nabla u_k) \geq \lambda\eta(r)g(v_k), \quad \text{div}(A_2(|\nabla v_k|)\nabla v_k) \geq \lambda\eta(r)g(u_k).$$

Let $h_1(r), h_2(r)$ be positive solutions of

$$\text{div}(A_1(|\nabla h_1|)\nabla h_1) = \lambda\eta(r)g(h_1), \quad 0 \leq r < R,$$
$$h_1 \to \infty, \quad r \to R_0^-,$$

and

$$\text{div}(A_2(|\nabla h_2|)\nabla h_2) = \lambda\eta(r)g(h_2), \quad 0 \leq r < R_0,$$
$$h_2 \to \infty, \quad r \to R_0^-,$$

where R_0 is an arbitrary positive real number. It is now easy to show by Lemma 2.3 that $u_k + v_k \leq h_1 + h_2$ in $[0, R_0]$. Hence $u + v = \lim_{k\to\infty}(u_k + v_k) \leq h_1 + h_2$ on $[0, R_0]$. Since R_0 is arbitrary, the functions u, v exist on $\mathbf{R}^N$ and hence are entire so that $(b_0, a_0) \in \mathcal{G}$. On the other hand, if $\max\{b_0, c_0\} = c_0$, then $C \leq c_0 - \frac{1}{k}$ for large k so that $v_k \geq C$ for all $r \geq 0$ and for all sufficiently large k. Then $u_k(r) \geq C^\alpha A(r)$ where

$$A(r) = \int_0^r \Phi_1(t^{1-N}\int_0^t s^{N-1}p(s)ds)dt$$

and the proof continues as before with C replaced by $C^\alpha A(r)$.

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Competing Interests

The authors declare that no competing interests exist.

References

[1] Yang Z.D. Existence of entire explosive positive radial solutions for a class of quasilinear elliptic systems, J.Math. Anal. Appl. 2003;288:768-783.

[2] Wei L, Yang Z.D. Large solutions of quasilinear elliptic systems of competitive type: existence and asymptotic behavior, Int. J. Diff. Eqns. 2010;2010:1-17.

[3] Yang Z.D, Lu Q.S. Existence of entire explosive radial solutions for a class of sublinear elliptic systems, Comm. Nonlinear. Sci. Numer. Simul. 2001;6:88-92.

[4] Yang Z.D, Lu Q.S. Blow-up estimates for a non-Newtonian filtration system, Appl. Math. Mech. 2001;22:287-294.

[5] Kuang J.C. Applied Inequalities, 4th Edition. Shandong Science and technology Press, Ji'nan(in Chinese). 2010.

[6] Reddy Narsimha K. Integral inequalities and applications, Bull. Austral. Math. Soc. 1980;21:13-20.

[7] Yang Z.D, Wu Z. Existence of boundary blow-up solutions for a class of quasilinear elliptic systems with subcritical case, Commun. Pure. Appl. Anal. 2007;6:531-540.

[8] Lair A.V, Wood A.W. Existence of entire large positive solutions of semilinear elliptic systems, J. Differential Equations. 2000;164:380-394.

[9] Yang Z.D, Lu Q.S. Non-existence of positive radial solutions for a class of quasilinear elliptic systems, Comm. Nonlinear. Sci. Numer. Simul. 2000;5:184-187.

[10] Clement Ph, Manasevich R, Mitidieri E. Positive solutions for a quasilinear systems via blow up, Comm. Partial Differential Equations. 1993;18:2071-2106.

[11] Guo Z.M. some existence and multiplicity results for a class of quasilinear elliptic eigenvalue problem, Nonlinear Anal. 1992;18:957-971.

[12] Garcia-Melian J, Rossi J.D. Boundary blow-up solutions to elliptic systems of competitive type, J. Differential Equations. 2004;206:156-181.

[13] Cirstea F.-C, Radulescu V. Entire solutions blowing up at infinity for semilinear elliptic sytems, J. Math. Pure Appl. 2002;81:827-846.

[14] Garcia-Melian J. A remark on uniqueness of large solutions for elliptic systems of competitive type, J. Math. Anal. Appl. 2007;331:608-616.

[15] Lair A.V. A necessary and sufficient condition for the existence of large solutions to sublinear elliptic systems, J. Math. Anal. Appl. 2010;365:103-108.

[16] Lair A.V. Entire large solutions to semilinear elliptic systems, J. Math. Anal. Appl. 2011;382:324-333.

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