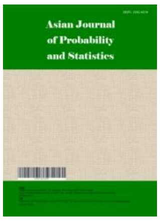

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Evaluation and Comparison of Three Classes of Central Composite Designs

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Authors' contributions

This work was carried out in collaboration among all authors. Author FCK-L designed the study, performed the statistical analysis, wrote the protocol and wrote the first draft of the manuscript. Authors LCK and DCB managed the analyses of the study. Authors CPO and AFC managed the literature searches. All authors read and approved the final manuscript.

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Abstract

Three classes of Central Composite Design (CCD): Central Composite Circumscribed Design (CCCD), Central Composite Inscribed Design (CCID) and Central Composite Face-Centered Design (CCFD) in Response Surface Methodology (RSM) were evaluated and compared using the A-, D-, and G-efficiencies for factors, k, ranging from 3 to 10**,** with 0-5 centre points, in other to determine the performances of the designs under consideration. The results show that the CCDs (CCCD, CCFD and CCID) are at their best when the G-efficiency is employed for all the factors considered while the CCID especially behaves poorly when using the A- and D-efficiencies.

Keywords: Response surface methodology; central composite circumscribed design; central composite inscribed design; central composite face-centered design; A-; D-; and G-Efficiencies.

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1 Introduction

In every field of inquiry, experiments are carried out by researchers in other to study the effects of some design attributes or variables and model them on the response of interest. The origination of response surface methodology (RSM) could be traced back to Box and Wilson [1]. RSM, can be said to be a set technique (Mathematical and Statistical) used empirically for model building and evaluation. In the application of RSM, the relationship between the dependent variable and the independent variables are investigated. Usually, the form of such relationship is not always unknown, but, it can be estimated through a second-order response surface model given in (1).

$$y = \beta_0 + \sum_{i=1}^{k} \beta_i x_i + \sum_{i=1}^{k} \beta_{ii} x_{ii}^2 + \sum_{j=i+1}^{k} \sum_{i=1}^{k-1} \beta_{ij} x_i x_j + \varepsilon_{ij} \tag{1}$$

where, y is the measured response or dependent variable, $\beta_0, \beta_1, ..., \beta_k$ are the model parameters , $x_1, x_2, ..., x_k$ are the independent variables and ε_{ij} is an error term.

The Central Composite designs most often utilize the stated model in (1). According to Myers and Montgomery [2] and Box and Draper [3], one of the most important criteria to be considered in the choice of second-order response surface design is the stability of the prediction variance over the region of interest. For examples of the type of second-order designs used in response surface exploration, see, Box and Wilson [1], Myers and Montgomery [2] and Zahran et al [4].

It is important to note the fact that a design performs better to other designs under certain optimality criterion does not always grantee that it will retain such performance when considered by other optimality criterions. Hence, to choose a design, attention will be on the choice of design evaluation criteria used. The commonly optimality criteria used in design evaluation are A-, D-, E- and G- optimality criteria. Atkinson and Donev [5], gave a comprehensive and detailed overview of design evaluation criteria. The CCD is one of the most popular response surface designs. And it has been wildly used and applied in RSM exploration for fitting the second-order model since its introduction by Box and Wilson [1]. The partial duplication of the factorial portion and partial duplication of the star portion of the CCDs were studied by Dykstra [6] for some factors *p, p = 2, 3… 8*. It was shown from the result that the star portion duplicated design are potentially better than the partially duplicated design. Lucas [7], evaluated four types of optimum composite design in different regions of interest. It was discovered form the result that symmetric composite designs are nearly optimum for experiments in a hypercube region. Myers [8], suggested optimal CCDs under several design criteria (orthogonality and rotatability). Three CCDs – (CCCD, CCID and CCFD) were evaluated and compared on the studied of Motor assembly through simulation, the results revealed that for efficient exploration experiment of the design, correct selection of these designs most be made, see, Xianfeng and Zhang [9]. Oyejola and Nwanya [10], compared and evaluated five types of CCDs using A-, D-, G- and I- design optimality criteria, for factors ranging from 3 – 6 with replicated star portions and increased centre points. It shown from their result that, D- and G-optimality criteria of the CCDs reduced when the star points are replicated, while the case is otherwise for the A-optimality criterion. However, the I-optimality did not show any relative change when considered for the CCDs. In a study by Chigbu and Ohaegbulem [11], they compare partially replicated cube and star portions of the rotatable and orthogonal CCD using the D-optimality design criterion, it was found that the D- optimal performs better when the cube portion is replicated than replicating the star portion. Lucas [12] studied and compared the performances of second-order response surface designs using the D- and G-efficiencies, and it was evident from the result that the CCD performs better than the other designs compared.

In this study, three classes of central composite design (CCD): central composite circumscribed design (CCCD), central composite inscribed design (CCID), and central composite face-centered (CCFD), are evaluated and compared using the A-, D- and G-efficiencies for factors, k = 3, 4, 5, 6, 7, 8, 9 and 10. To reduce the number of design runs, which increase rapidly as the number of factors increases, especially from k = 5, the full factorial portions of the CCDs are employed for factors k = 3 and 4, while fraction of the factorial portions of the CCDs are employed for factors k = 5, 6,7,8,9 and 10. The performances of these designs were considered for centre point, n_0, ranging from 0 to 5.

2 Methodology

We present the three Central Composite Designs as well as the optimality criteria, which will be used to assess the designs under consideration.

2.1 Designs for comparison

The three CCDs: CCCD, CCID and CCFD were examined, evaluated and compared in terms of their A-, D- and G-efficiencies.

2.2 Central composite design

The CCD has gained popularity and wilder application since its introduction by Box and Wilson [1]. Assuming $k \geq 2$ design variables, the CCD consists of the following:

1. An $f = 2^{k-p}$ full ($p = 0$) or fractional ($p > 0$) factorial design of at least Resolution V; each point is of the form ($x_1, ..., x_k$) = ($\pm 1, \pm 1, ..., \pm 1$);
2. $2k$ star points, of the form ($x_1, ..., x_i, ..., x_k$) = ($0, ..., 0, \pm \alpha, 0, ..., 0$), for $1 \leq i \leq k$;
3. n_0 replication of the centre points ($x_1, ..., x_k$) = ($0, 0, ..., 0$): see, for example, Onukogu and Chigbu [13], pp 72-73.

In other to reduce the number of design runs in this work, which increases rapidly as the number of factors increases, especially from k = 5.The full factorial portions of the CCDs are employed for factors k = 3 and 4, while fraction of the factorial portions of the CCDs are employed for factors k = 5, 6,7,8,9 and 10. The performances of these designs were considered for centre point, n_0, ranging from 0 to 5. Let N denote the total number of runs in the CCD, $N = f + 2k + n_0$. where f is the number of factorial points, $2k$ is the number of axial points and n_0, the number of centre points. The choice and values of axial distance, α, is based on the region of interest and the type CCD. The α, considered in this work is the rotatable α, for CCCD and CCID. While for the CCFD, $\alpha = \pm 1$ was considered, since the star points are at the centre of each face of the factorial space.

The structure of the CCD design matrix, X, for any two input variables, x_i and x_j, with one centre point is given as:

$$
\begin{array}{rcl}
 & \begin{matrix} x_0 & x_i & x_j & x_i x_j & x_i^2 & x_j^2 \end{matrix} & \} \ \textit{points} \\
X = & \begin{bmatrix}
1 & 1 & 1 & 1 & 1 & 1 \\
1 & 1 & -1 & -1 & 1 & 1 \\
1 & -1 & 1 & -1 & 1 & 1 \\
1 & -1 & -1 & 1 & 1 & 1 \\
1 & \alpha & 0 & 0 & \alpha^2 & 0 \\
1 & -\alpha & 0 & 0 & \alpha^2 & 0 \\
1 & 0 & \alpha & 0 & 0 & \alpha^2 \\
1 & 0 & -\alpha & 0 & 0 & \alpha^2 \\
1 & 0 & 0 & 0 & 0 & 0
\end{bmatrix} &
\begin{array}{l}
\left.\begin{matrix} \\ \\ \\ \\ \end{matrix}\right\} f = 2^k = 4 \\
\left.\begin{matrix} \\ \\ \\ \\ \end{matrix}\right\} 2k = 4 \\
\} \ n_0 = N - f - 4
\end{array}
\end{array}
$$

2.3 Central composite circumscribed design

The central composite circumscribed design (CCCD) is in the original structure of the CCD, with the star points located at the centre, at some distance, α. For all the factors, the star points establish extremes for the low and high settings. These designs require five levels for each factor. Augmenting a two-level factorial or fraction (resolution V) with a $2k$ axial or star points and centre points can produce this design. The matrix structure of the central composite circumscribed design for $k = 3$, with $n_0 = 1$ and $\alpha = (f)^{\frac{1}{4}}$ is given as:

$$
\begin{array}{c}
\begin{array}{cccc} x_0 & x_1 & x_2 & x_3 \end{array} \\
\begin{bmatrix}
1 & -1 & -1 & -1 \\
1 & -1 & -1 & 1 \\
1 & -1 & 1 & -1 \\
1 & -1 & 1 & 1 \\
1 & 1 & -1 & -1 \\
1 & 1 & -1 & 1 \\
1 & 1 & 1 & -1 \\
1 & 1 & 1 & 1 \\
1 & -1.6818 & 0 & 0 \\
1 & 1.6818 & 0 & 0 \\
1 & 0 & -1.6818 & 0 \\
1 & 0 & 1.6818 & 0 \\
1 & 0 & 0 & -1.6818 \\
1 & 0 & 0 & 1.6818 \\
1 & 0 & 0 & 0
\end{bmatrix}
\end{array}
$$

2.4 Central composite inscribed design

The CCID is a scaled down CCCD, with each factor level of the CCCD divided by, α, to generate the CCID. This design, being a scaled down of the CCCD also requires five levels of each factor, because the star points lie within the space of the factorial design. The matrix structure of the central composite inscribed design for $k = 3$ with $n_0 = 1$ is given as:

$$
\begin{array}{c}
\begin{array}{cccc} x_0 & x_1 & x_2 & x_3 \end{array} \\
\begin{bmatrix}
1 & -0.5946 & -0.5946 & -0.5946 \\
1 & -0.5946 & -0.5946 & 0.5946 \\
1 & -0.5946 & 0.5946 & -0.5946 \\
1 & -0.5946 & 0.5946 & 0.5946 \\
1 & 0.5946 & -0.5946 & -0.5946 \\
1 & 0.5946 & -0.5946 & 0.5946 \\
1 & 0.5946 & 0.5946 & -0.5946 \\
1 & 0.5946 & 0.5946 & 0.5946 \\
1 & -1 & 0 & 0 \\
1 & 1 & 0 & 0 \\
1 & 0 & -1 & 0 \\
1 & 0 & 1 & 0 \\
1 & 0 & 0 & -1 \\
1 & 0 & 0 & 1 \\
1 & 0 & 0 & 0
\end{bmatrix}
\end{array}
$$

2.5 Central composite face-centered design

The CCFD is a special case of a CCD, in which $\alpha = 1$. As a result, the CCFD becomes a three-level design, because the star point is located at the centre of the face of the cube, requiring three levels for each face. The axial and the factorial points of face-centered CCD fall onto the surface of the cube. The matrix structure of the central composite face-centered design for $k = 3$ with $n_0 = 1$ is given as:

$$\begin{array}{cccc} x_0 & x_1 & x_2 & x_3 \end{array}$$
$$\begin{bmatrix} 1 & -1 & -1 & -1 \\ 1 & -1 & -1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & -1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & 1 & -1 & 1 \\ 1 & 1 & 1 & -1 \\ 1 & 1 & 1 & 1 \\ 1 & -1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & -1 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & -1 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

Two of these designs, CCCD and CCID, have a common characteristic; they are rotatable.

According to Box and Hunter [14], a rotatable design is one in which the estimated response, y, has a constant variance at all points with the same distance from the centre of the design. For a CCD to be rotatable, $\alpha = (f)^{\frac{1}{4}}$, where f is the number of runs in the factorial portion of the CCD.

2.6 Optimality criteria

An optimal design is an experimental design that is based on a particular optimality criterion. Kiefer [15] detailed the theory behind optimum designs, which states that, if χ is a compact space on which the real function, f_t, are continuous and linearly independent, the probability measure, ξ, is D-optimum for an unknown m-vector, $\theta_1,...,\theta_m$ if and only if it is G-optimal. Design optimality criterion could be alphabetic because they are represented by the first letters of the names of the criteria. There commonly used design optimality criteria are the A-, D- and G-optimality criteria. Design efficiencies are computed in other to compare designs. The A-, D- and G-efficiencies are used in this work.

2.7 A-optimality criterion

This criterion, introduced by Chernoff [16], is an approach in which the inverse of the information matrix, $\left(X^{'}X\right)$ in other to to minimize the trace of the matrix. Using this also, the average variance of the estimates of the regression coefficients is minimized. The criterion is given by:

A-criterion = min $trace\left[\left(X^{'}X\right)^{-1}\right]$, and

The A-efficiency = $100\dfrac{p}{trace[N(X'X)^{-1}]}$.

The size of the design and the number of parameters in the model are N , p respectively.

2.8 D-Optimality criterion

D-optimality criterion is very easy to compute and has gained popularity among researchers since its introduction by Wald [17]. The D-optimality criterion utilizes the attributes of the moment matrix, M , to estimation of model parameters. The criterion is defined as,

$$M = \left[\frac{X'X}{N}\right],$$

The D-optimality criterion seeks to maximize the determinant of the information matrix, $X'X$, or equivalently seeks to minimize the inverse of the information matrix. That is, Max $\left|X'X\right|$ or Min $\left(X'X\right)^{-1}$, respectively.

D-efficiency of the criterion is given by = $100\frac{|X'X|^{1/p}}{N}$.

2.9 G-optimality criterion

This criterion is a prediction variance based optimality criterion. It utilizes the variance profile to make prediction at a particular location in the design space, or throughout the design region. According to Box and Hunter [14], this is achievable through variance function (the Scaled Prediction Variance), which is given by:

$$\frac{\mathrm{NVar[y(x)]}}{\sigma^2} = Nf'(x)(X'X)^{-1}f(x) \tag{4}$$

where, $f(x)$ is the vector coordinates of points in the region of interest, that is, $f'(x)$ = [1, x_1, …, x_k , x_1^2, …, x_k^2, x_1x_2, …, $x_{k-1}x_k$], where, N, X and σ^2 are the total sample size, design matrix and process variance of the design under consideration. A G-optimal design is one that minimizes the maximum SPV over the experimental design region. Symbolically, it is written as

$$\min\{\max N \operatorname{var} y(x)\} = \min\{N \max f'(x)(X'X)^{-1} f(x)\} \tag{5}$$

The G-efficiency = $100\frac{p/N}{\sigma^2_{\max}}$.

where, p and N are as defined before, and $\sigma^2_{\max}$ is the maximum of $f'(x)(X'X)^{-1}f(x)$: see Borkowski and Valeroso [18].

3 Comparison of the designs

In this section, the three classes of CCD (CCCD, CCID and CCFD) for factors $3 \le k \le 10$ are compared using the optimality criteria.

3.1 Design comparison using optimality criteria

In this section, the A-, D- and G-efficiencies of the three designs considered will be compared, the result will also be shown graphically. Let n_0 indicate the number of centre points and N the number of design runs.

The expanded design matrix for CCCD for $k = 3$ with $n_0 = 0$ is given by:

$$X = \begin{bmatrix} 1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & -1 & -1 \\ 1 & -1 & 1 & -1 & 1 & 1 & 1 & -1 & 1 & -1 \\ 1 & -1 & 1 & 1 & 1 & 1 & 1 & -1 & -1 & 1 \\ 1 & 1 & -1 & -1 & 1 & 1 & 1 & -1 & -1 & 1 \\ 1 & 1 & -1 & 1 & 1 & 1 & 1 & -1 & 1 & -1 \\ 1 & 1 & 1 & -1 & 1 & 1 & 1 & 1 & -1 & -1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & -1.68 & 0 & 0 & 2.83 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1.68 & 0 & 0 & 2.83 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & -1.68 & 0 & 0 & 2.83 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1.68 & 0 & 0 & 2.83 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & -1.68 & 0 & 0 & 2.83 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1.68 & 0 & 0 & 2.83 & 0 & 0 & 0 \end{bmatrix}$$

$$(X'X)^{-1} = \begin{bmatrix} 84.93 & 0 & 0 & 0 & -29.0 & -29.0 & -29.0 & 0 & 0 & 0 \\ 0 & 0.07 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.07 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.07 & 0 & 0 & 0 & 0 & 0 & 0 \\ -29.0 & 0 & 0 & 0 & 9.95 & 9.89 & 9.89 & 0 & 0 & 0 \\ -29.0 & 0 & 0 & 0 & 9.89 & 9.95 & 9.89 & 0 & 0 & 0 \\ -29.0 & 0 & 0 & 0 & 9.89 & 9.89 & 9.95 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.13 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.13 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.13 \end{bmatrix}$$

X = [1 1 1 1 1 1 1 1 1 1],
$x'(X'X)x$ = 0.7197;
then the scaled prediction variance, $Var(x)$, is given by
$Var(x) = Nx'(X'X)^{-1}x$ = 14(0.7197) = 10.0758;
And the G-efficiency is obtained as $100\frac{10/14}{0.7197} = 99.2477.$

The same procedure was used to obtain the scaled prediction variance and G- efficiencies for the CCCD, CCID and CCFD for different factors and different centre points.

The D-efficiency is obtained as: $100\left[\frac{|X'X|^{1/p}}{N}\right]$

$$X'X = \begin{bmatrix} 14.0000 & 0 & 0 & 0 & 13.6569 & 13.6569 & 13.6569 & 0 & 0 & 0 \\ 0 & 13.6569 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 13.6569 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 13.6569 & 0 & 0 & 0 & 0 & 0 & 0 \\ 13.6569 & 0 & 0 & 0 & 24.0000 & 8.0000 & 8.0000 & 0 & 0 & 0 \\ 13.6569 & 0 & 0 & 0 & 8.0000 & 24.0000 & 8.0000 & 0 & 0 & 0 \\ 13.6569 & 0 & 0 & 0 & 8.0000 & 8.0000 & 24.0000 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 8.0000 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 8.0000 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 8.0000 \end{bmatrix}$$

$Det(X'X) = 1.5725e+008.$

$$D-efficiency = 100\left[\frac{(1.5725e+008)^{1/10}}{14}\right] = 47.1554.$$

A-efficiency is obtained as: $100\left[\frac{p}{trace[N(X'X)^{-1}]}\right].$

$$N[(X'X)^{-1}] =$$

$$1.0e+003\begin{bmatrix} 1.1890 & 0 & 0 & 0 & -0.4059 & -0.4059 & -0.4059 & 0 & 0 & 0 \\ 0 & 0.0010 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.0010 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.0010 & 0 & 0 & 0 & 0 & 0 & 0 \\ -0.4059 & 0 & 0 & 0 & 0.1393 & 0.1384 & 0.1384 & 0 & 0 & 0 \\ -0.4059 & 0 & 0 & 0 & 0.1384 & 0.1393 & 0.1384 & 0 & 0 & 0 \\ -0.4059 & 0 & 0 & 0 & 0.1384 & 0.1384 & 0.1393 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.0018 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.0018 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.0018 \end{bmatrix}$$

$trace[N(X'X)^{-1}] = 1.6152e+003.$

$$A-efficiency = 100\left[\frac{10}{1.6152e+003}\right] = 0.6191.$$

Using the same procedure, the following results, presented in Tables 1 to 8, are obtained for CCCD, CCID and CCFD for 3 to 10 factors with 0 to 5 centre points for each factor.

Table 1. Summary Statistics for the Three Classes of CCD for k = 3

Design	n_o	N	D-efficiency	G-efficiency	A-efficiency
CCCD	0	14	47.1554	99.2477	0.6191
	1	15	68.7038	67.4491	32.0718
	2	16	68.9923	93.2836	44.1517
	3	17	67.6076	87.8094	49.3150
	4	18	65.7088	82.9435	51.3877
	5	19	63.6514	78.5781	51.9060
CCID	0	14	9.9132	99.2477	0.2182
	1	15	14.4432	67.4491	7.7747
	2	16	14.5038	93.2836	9.3158
	3	17	14.2127	87.8094	9.6713
	4	18	13.8136	82.9435	9.6318
	5	19	13.3810	78.5781	9.4338
CCFD	0	14	46.3045	89.2856	31.0559
	1	15	44.7163	83.6260	31.2908
	2	16	42.9990	78.5472	30.6813
	3	17	41.2965	74.0198	29.7416
	4	18	39.6635	69.9692	28.6826
	5	19	38.1206	66.3200	27.5998

Table 2. Summary Statistics for the Three Classes of CCD for k = 4

Design	n_o	N	D-efficiency	G-efficiency	A-efficiency
CCCD	0	24	0	0	0
	1	25	76.7266	60	31.6484
	2	26	77.2647	98.9068	45.3972
	3	27	76.4417	95.2435	55.2876
	4	28	75.1389	91.8420	55.9007
	5	29	73.6354	88.6750	57.7385
CCID	0	24	0	0	0
	1	25	8.3493	60	4.5000
	2	26	8.4079	98.9068	5.3254
	3	27	8.3183	95.2435	5.5556
	4	28	8.1765	91.8420	5.5901
	5	29	8.0129	88.6750	5.5419
CCFD	0	24	45.7448	94.7400	25.9366
	1	25	44.5232	90.9918	25.4855
	2	26	43.3015	87.5319	24.9074
	3	27	42.1055	84.3028	24.2751
	4	28	40.9479	81.3167	22.0506
	5	28	39.8347	78.5246	22.9789

where Infinity implies that the D- and G-efficiencies are at infinity.

Table 3. Summary Statistics for the Three Classes of CCD for k = 5

Design	n_o	N	D-efficiency	G-efficiency	A-efficiency
CCCD	0	26	70.0500	90.1644	15.3238
	1	27	72.4642	88.1934	40.4332
	2	28	71.8171	85.2079	49.8270
	3	29	70.5492	82.3352	53.9832
	4	30	69.0660	79.6178	55.8140
	5	31	67.5089	77.0758	56.4569
CCID	0	26	6.9498	90.1644	2.7535
	1	27	7.1893	88.1934	4.6823
	2	28	7.1252	85.2079	4.9931
	3	29	6.9994	82.3352	5.0292
	4	30	6.8522	79.6178	4.9763
	5	31	6.6977	77.0758	4.8875
CCFD	0	26	44.0194	83.6727	25.9398
	1	27	42.6900	80.5904	25.2012
	2	28	41.4195	77.7283	24.4658
	3	29	40.2099	75.0558	23.7483
	4	30	39.0602	72.5614	23.0558
	5	31	37.9682	70.2280	22.3915

Table 4. Summary Statistics for the Three Classes of CCD for k = 6

Design	n_o	N	D-efficiency	G-efficiency	A-efficiency
CCCD	0	44	73.6955	99.6654	1.5714
	1	45	81.8114	64.0542	34.0467
	2	46	81.9967	96.4958	48.0542
	3	47	81.4085	94.4427	55.3616
	4	48	80.5287	92.4752	59.5174
	5	49	79.5123	90.5879	61.9579
CCID	0	44	3.7784	99.6654	0.2662

Design	n_o	N	D-efficiency	G-efficiency	A-efficiency
	1	45	4.1946	64.0542	2.5171
	2	46	4.2040	96.4958	2.8168
	3	47	4.1739	94.4427	2.8986
	4	48	4.1288	92.4752	2.9136
	5	49	4.0767	90.5879	2.9006
CCFD	0	44	45.6289	94.0671	19.3190
	1	45	44.7976	91.9903	18.9792
	2	46	43.9848	89.9905	18.6377
	3	47	43.1923	88.0758	18.2987
	4	48	42.4213	86.2409	17.9649
	5	49	41.6723	84.4934	17.6379

Table 5. Summary Statistics for the Three Classes of CCD for k = 7

Design	n_o	N	D-efficiency	G-efficiency	A-efficiency
CCCD	0	78	87.3555	76.9231	8.2834
	1	79	90.4324	55.6950	32.7157
	2	80	90.7973	81.8182	46.1538
	3	81	90.6068	81.0586	54.4337
	4	82	90.1763	80.2018	59.8893
	5	83	89.6175	79.3224	63.6365
CCID	0	78	2.2955	76.9231	0.6767
	1	79	2.3764	55.6950	1.3936
	2	80	2.3860	81.8182	1.5437
	3	81	2.3810	81.0586	1.5985
	4	82	2.3697	80.2018	1.6203
	5	83	2.3550	79.3224	1.6270
CCFD	0	78	46.4779	89.5843	13.0056
	1	79	46.0085	89.5629	12.8772
	2	80	45.5411	89.3744	12.7464
	3	81	45.0771	89.0670	12.6144
	4	82	44.6175	88.6560	12.4822
	5	83	44.1631	88.1934	12.3504

Table 6. Summary Statistics for the Three Classes of CCD for k = 8

Design	n_o	N	D-efficiency	G-efficiency	A-efficiency
CCCD	0	80	41.7017	93.7500	0
	1	81	87.8689	55.5556	32.3232
	2	82	88.1446	99.7783	47.4621
	3	83	87.8708	98.5761	55.9658
	4	84	87.3816	97.4026	61.2245
	5	85	86.7829	96.2567	64.6610
CCID	0	80	1.0343	93.7500	0
	1	81	2.1794	55.5556	1.3419
	2	82	2.1863	99.7783	1.4872
	3	83	2.1795	98.5761	1.5315
	4	84	2.1673	97.4026	1.5461
	5	85	2.1525	96.2567	1.5480
CCFD	0	80	47.3635	97.9453	13.5431
	1	81	46.8607	96.7361	13.3985
	2	82	46.3644	95.5564	13.2544
	3	83	45.8751	94.4051	13.1113
	4	84	45.3932	93.2813	12.9697
	5	85	44.9189	92.1838	12.8297

Table 7. Summary Statistics for the Three Classes of CCD for k = 9

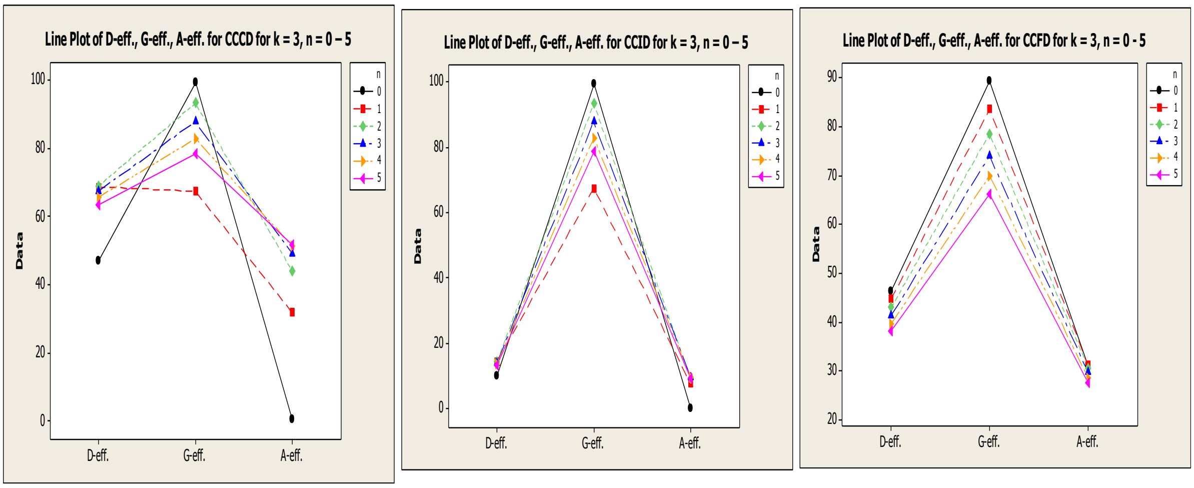
Design	n_o	N	D-efficiency	G-efficiency	A-efficiency
CCCD	0	146	95.1961	65.5038	24.6728
	1	147	95.7713	67.7685	40.1483
	2	148	95.8360	68.2501	49.9540
	3	149	95.6959	68.2558	56.7398
	4	150	95.4467	68.0777	61.6512
	5	151	95.1310	67.8158	65.3207
CCID	0	146	1.2082	65.5038	0.6807
	1	147	1.2155	67.7685	0.7802
	2	148	1.2163	68.2501	0.8161
	3	149	1.2145	68.2558	0.8326
	4	150	1.2114	68.0777	0.8406
	5	151	1.2073	67.8158	0.8443
CCFD	0	146	48.0001	74.1998	8.5040
	1	147	47.7326	74.2360	8.4560
	2	148	47.4650	74.2056	8.4075
	3	149	47.1978	74.1220	8.3586
	4	150	46.9314	73.9993	8.3096
	5	151	46.6660	73.8521	8.2605

Table 8. Summary statistics for the Three Classes of CCD for k = 10

Design	n_o	N	D-efficiency	G-efficiency	A-efficiency
CCCD	0	148	92.2430	77.5423	10.4567
	1	149	93.7285	57.0375	33.9412
	2	150	93.9179	81.6781	47.1633
	3	151	93.8100	81.3789	55.5256
	4	152	93.5685	80.9792	61.2067
	5	153	93.2527	80.5250	65.2541
CCID	0	148	1.1202	77.5423	0.4616
	1	149	1.1382	57.0375	0.7301
	2	150	1.1405	81.6781	0.7836
	3	151	1.1392	81.3789	0.8037
	4	152	1.1363	80.9792	0.8125
	5	153	1.1324	80.5250	0.8162
CCFD	0	148	49.3425	87.8365	8.9235
	1	149	49.0560	87.2470	8.8706
	2	150	48.7707	87.6145	8.9369
	3	151	48.4871	87.4347	8.7649
	4	152	48.2052	87.2259	8.7122
	5	153	47.9252	86.9878	8.6598

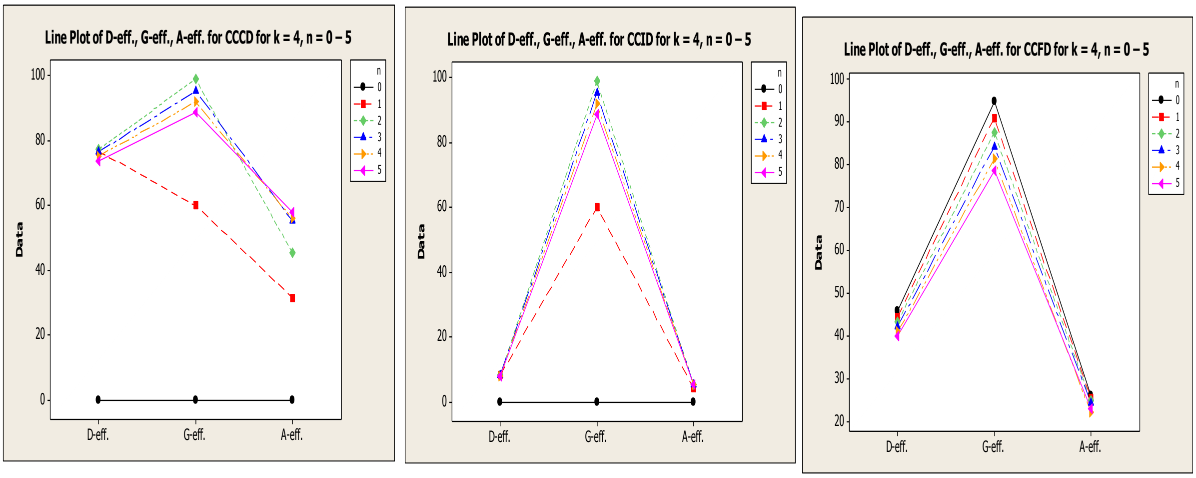
3.2 Graphical presentation of results and discussion

Graphical presentation of results in Fig. 1 – 8, for factors k = 3 – 10 and with centre points 0 -5

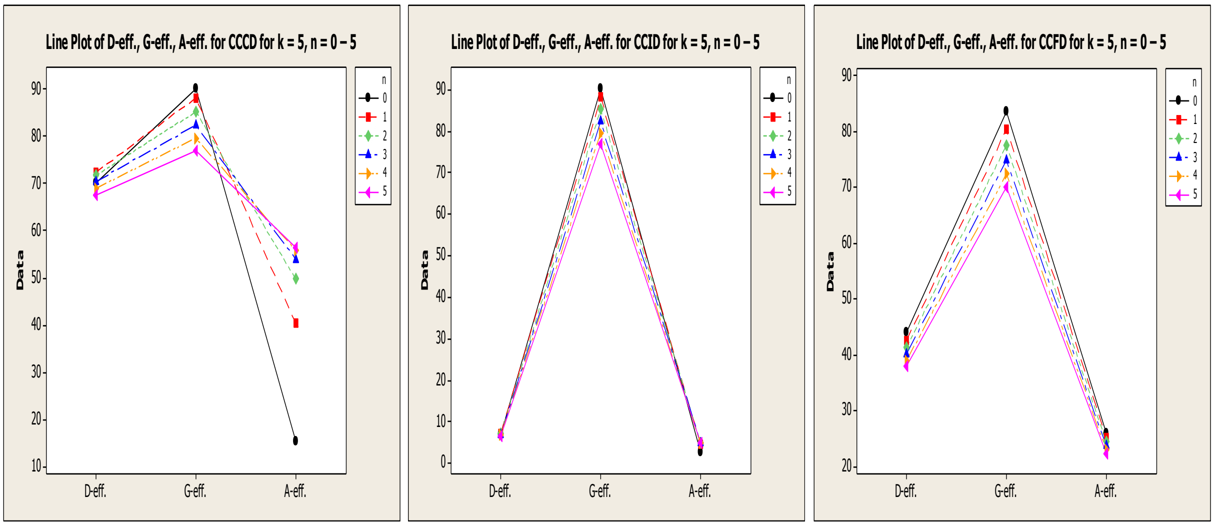


Three-Factor Design: Fig. 1 shows that with $n_0 = 0$, the D-efficiency values for the CCCD and CCID are low; an increase in n_0 to 1 and 2, increases the D-efficiency values, and decreases as n_0 increases. For CCFD, the D-, G- and A-efficiency values are high with $n_0 = 0$, but reduce as n_0 increases.

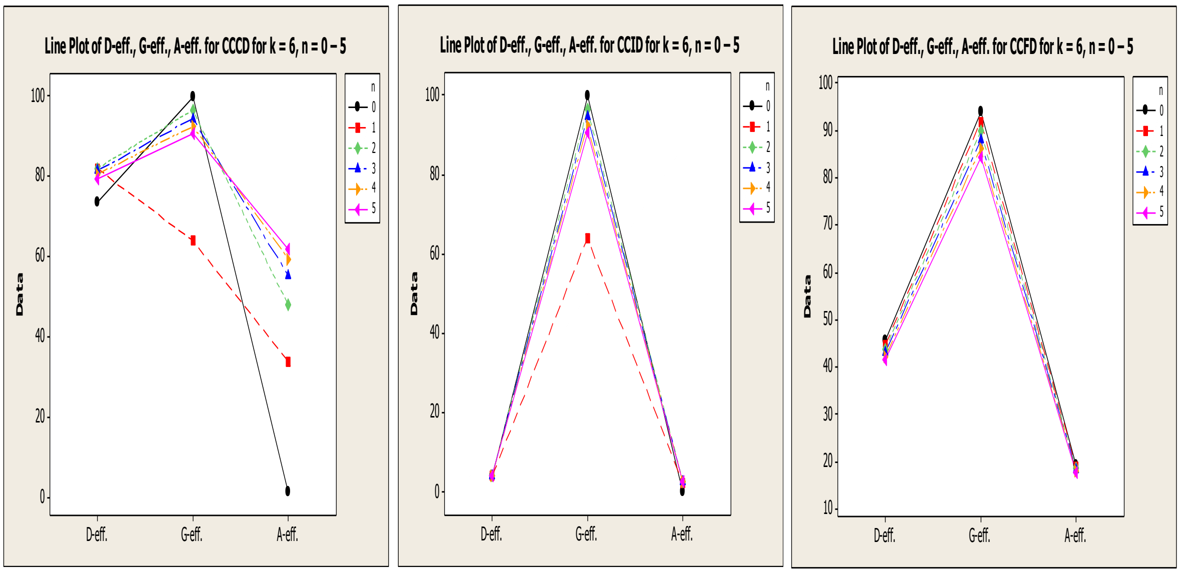
For the G-efficiency, the CCCD and CCID tend to have high values with $n_0 = 0$, reduce with $n_0 = 1$, increase with $n_0 = 2$, and thereafter, an increase of the n_0 reduces the G-efficiency value



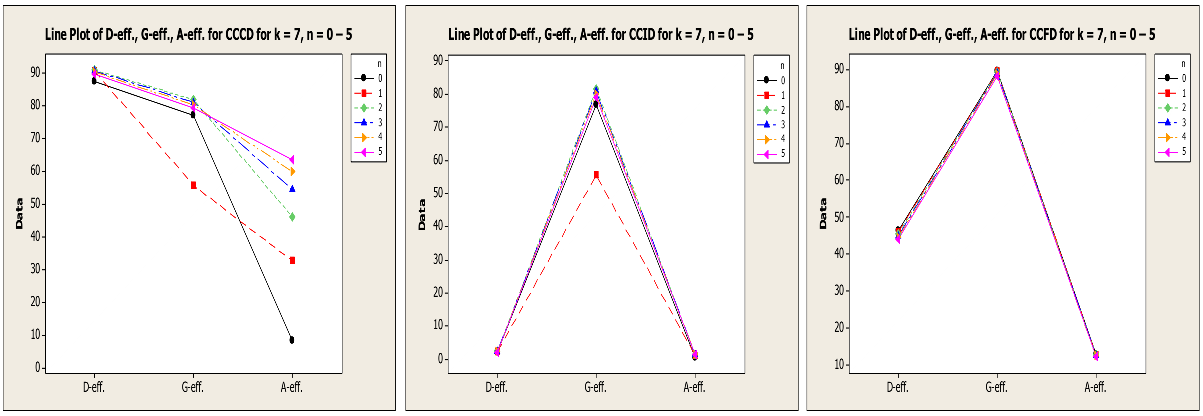
Four-Factor Design: Fig. 2. shows that the D- and G-efficiency values for the CCCD and CCID with $n_0 = 0$, are at infinity. Increasing n_0 tends to fluctuate the D- and G-efficiency values for the CCCD and CCID; an increase of n_0 increases the A-efficiency values for the CCCD. The D- and A-efficiency values for the CCID from $n_0 = 1 \text{ to } 5$ are relatively the same. For the CCFD, the D-, G- and A-efficiency values tend to reduce with an increase in n_0



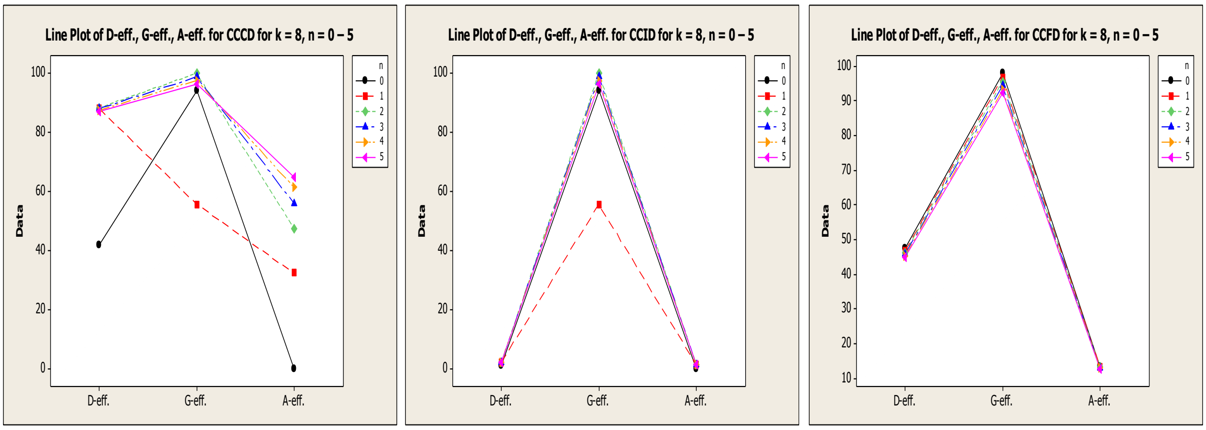
Five-Factor Design: Fig. 3. shows that increasing n_0 reduces the D-, G- and A-efficiency values of the CCFD. At $n_0 = 0,$ the G-efficiency values of the CCCD and CCID tend to be high, which reduces as n_0 increases. The D- and A-efficiency values of the CCCD fluctuate as n_0 increases



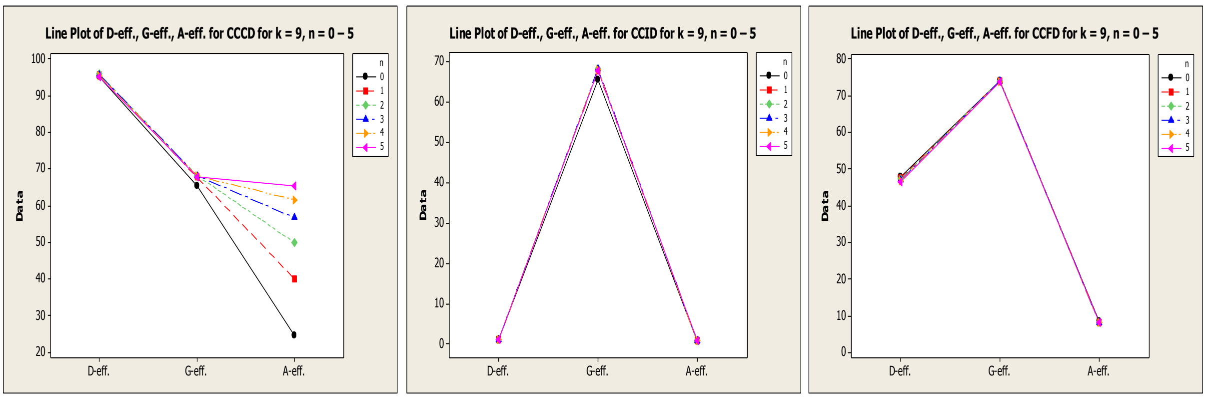
Six-Factor Design: Fig 4. shows that increasing n_0 increases the D- and A-efficiency values for the CCCD and CCID, but for the CCCD, the D-efficiency values started to decline at $n_0 = 3$. Also at $n_0 = 0$, the G-efficiency values for the CCCD and CCID are high; thereafter, it begins to fluctuate. For the CCFD, the D-, G- and A-efficiency values reduce as n_0 increases



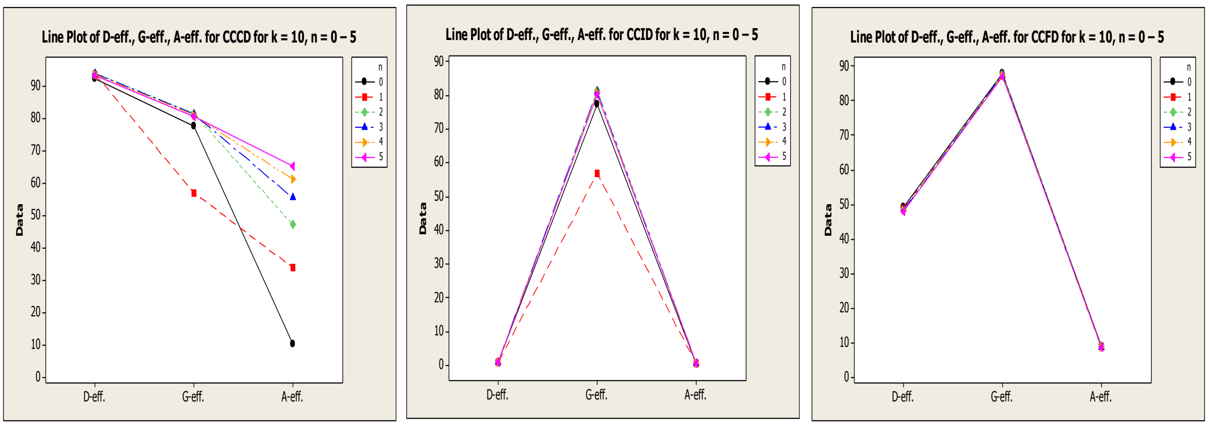
Seven-Factor Design: Fig 5 shows the same conclusion as in the case of the six-factor design



Eight-Factor Design: Fig. 6. Shows the same conclusion as in the case of six-factor design, only that the A-efficiency values for the CCCD and CCID at $n_0 = 0$ is 0



Nine-Factor Design: Fig. 7 shows an almost equal D- and G-efficiency values for the CCCD and CCID while the A-efficiency values for both CCCD and CCID increases as n_0 increases. For the CCFD, the D-, G- and A-efficiency values tend to reduce slightly with an increase in n_0.



Ten-Factor Design: Fig. 8. Shows slightly equal D-efficiency values for the CCCD and CCID. Also, the A-efficiency values for the CCCD and CCID tend to increase as n_0 increases. However, their G-efficiency values fluctuate with increase in n_0. For the CCFD, Increasing n_0 tend to reduce the D-, G- and A-efficiency values

4 Findings and Conclusions

4.1 Findings

Three classes of central composite design, namely: Central Composite Circumscribed Design, Central Composite Inscribed Design and Central Composite Face-Centered Design are compared for factors, k, ranging from 3 to 10 with 0 – 5 centre points, respectively, using the D -, G- and A-efficiencies. The results show that the CCDs perform better when the G-efficiency is employed for all the factors considered. Also increasing the centre points tend to reduce the D-, G- and A-efficiency values of the CCFD. The CCCD and CCID behave alike in terms of the G-efficiency criterion; the CCCD performs better than the CCID and CCFD when the D- and A-efficiency criteria are employed, but with centre points greater than zero.

4.2 Conclusion

From the foregoing, it can be seen that for factors k = 3, 4, 5, 6 and 8, the G- efficiency performs better than the D- and A- efficiencies for the number of parameter, N, and the number of centre points n_0 considered. For factor k = 7, 9 and 10 the D- efficiency performs better than the G- and A-efficiencies for CCCD, while the G-efficiency performs better than the D- and A- efficiencies for CCID and CCFD, for the number of parameter, N, and the number of centre points n_0 considered.

In general the CCDs give high efficiency values when the G-efficiency is employed and it can also be seen that the CCID and CCFD have low efficiencies values under the D- and A- efficiencies respectively for the number of parameter, N, and the number of centre points n_0 considered. Finally, the CCCD performs better than the CCID and CCFD when the D- and A-efficiency criteria are employed, but with centre points greater than zero which implies that the CCCD is a better CCD, when compared but the inclusion of centre points is recommended.

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Competing Interests

Authors have declared that no competing interests exist.

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