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Simulation of Steady State Heat Conduction Using an Hybrid Floating Walk and Markov Chain Technique

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Author's contribution

The sole author designed, analyzed and interpreted and prepared the manuscript.

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ABSTRACT

Markov chain technique had been developed for numerical simulation of steady state heat conduction. However the technique could only be used to handle domain with regular boundaries. An hybrid of floating walk and Markov chain techniques has therefore been developed.
The technique was used to simulate temperature distribution in rectangular and two arbitrary shaped domains with mixed boundary conditions. The results obtained were compared with that obtained using finite difference and as well as using floating walk technique. Results were statistically analysed using ANOVA (α =0.05) and the computer execution for all the three cases considered compared.
The results from the developed hybrid technique were not significantly different from those from finite difference and floating walk techniques. The hybrid technique execution time was longer than that of finite difference technique but shorter than the floating walk technique.
The study established the suitably of the hybrid floating walk markov chain technique for analysis of steady state heat conduction of arbitrary shaped domain.

Keywords: Heat conduction; arbitrary domain; numerical technique.

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1. INTRODUCTION

Several numerical techniques to model heat conduction have been documented. Finite difference technique was used by Anjo [1] to simulate solidification of aluminium in green sand mould while Dhawan and Kumar [2] reported the use of finite element for heat transfer analysis. The study investigated the effect of different combinations of meshes and numerical schemes for modelling the cooling of an aluminium plate. Takemori et al. [3] carried out a numerical simulation of welding process in the development of a new compressor. The numerical results were supported with experimental studies. Yavuzturk et al. [4] modelled U Tube Ground heat exchanger using finite volume technique while Motley and Provost [5] developed a mapped infinite mapped element approach for a one dimensional transient heat conduction. Shen et al. [6] used a finite difference technique to model heat transfer during a grinding process. An optimization technique was developed by Mehta [7] to calculate for temperature in a finite conducting slab. The result was reported to compare well with finite difference solution. A fuzzy finite element was used to analyse steady state heat conduction by Majumdar [8]. The technique allowed more accurate estimation of material properties used in the analysis; however only 1-dimensional case was considered. Kovranyuk and Chebotarev [9] investigated a combined radiative-conductive- convective phenomenon. A weak solution was obtained which was used to carry out some numerical experiments.

Monte Carlo technique which is a probability technique was first reported by Haji-Sheikh and Sparrow [10] for modelling heat conduction. The method has not received much attention due to its slow execution time although with capability to determine solution at a point much easily. Other probability approaches include heat conduction analysis by Grigorin [11] and thermal studies of a superconductivity current limiter by Leveque and Rezzong [12]. Two main versions of Monte Carlo technique namely fixed and floating walk have been developed. The fixed walk has a very slow execution time and could not easily handle irregular boundary cases. Floating walk has comparatively fast execution time than the fixed walk and could easily handle irregular boundary.

A Markov chain technique was developed by Sadiku et al. [13], the executive time was fastly improved; it was however applied to simple steady state heat conduction cases such as a slab with isothermal boundaries and is thus restrictive in application to only surfaces with regular boundaries. Although Cho [14] discussed the application of particle transport Monte Carlo technique to heat conduction analysis of arbitrary geometries, cases considered were limitive. Ravichandran and Minnic [15] only treated boundary conditions associated with heat conduction at nanoscale and with little consideration for numerical analysis.

The objective of this work is to develop a numerical technique which could be used to analyse heat conduction in a 2- dimensional domain at a reasonably fast pace. Thus a hybrid of floating walk (which has capability of handling irregular boundary) and Markov chain (with lower execution time than floating walk) techniques is to be developed for analysis of steady state heat conduction. It is premised that the developed technique would be able to handle analysis of steady state heat conduction in any domain at a fast rate by utilizing both the inherent capabilities in the floating walk and Markov chain techniques. The developed technique will be used to analyse heat conduction in an arbitrary shaped slab and as well as rectangular slab with non isothermal boundaries.

2. DESCRIPTION OF THE METHODOLOGY

In this section, the floating walk and regional Markov Chain techniques are first discussed, thereafter the developed hybrid technique is described.

2.1 Floating Walk Technique

Steady state heat conduction in a slab with internal heat generation could be presented as:

$$\frac{\partial^2 T}{\partial^2 x} + \frac{\partial^2 T}{\partial^2 y} + \frac{q}{k} = 0 \quad (1)$$

with T representing temperature, q, the heat generating source and k, the thermal conductivity of the material. If q is zero, the exact solution at location (x,y) for an homogeneous 2-dimensional domain with radius r as recast by Oluwajobi and Jeje [16] could be represented as

$$T(x,y) = \int_0^1 T(r,\omega) dF\omega \quad (2)$$

where φ is angular displacement and

$$F = \frac{\omega}{2\pi} \quad (3)$$

Eqn.(2) assumed a normal distribution curve. It suggests that the temperature at any location (x,y) is an average of all the temperature values at the periphery of the domain with radius *r.* Haji-Sheikh and Sparrow [10] based on this concept developed a floating walk technique(a probability method) that was subsequently applied to evaluate temperature distribution in a steady state heat conduction in a solid domain. The basic concept of the floating walk is described below.

In a given domain, a walk is commenced by a fictitious particle at the point where the solution is being sought. The distance between the particle and the boundary (r_i) is determined. Thereafter the walk is commenced by selecting a random number, *F* between 0 and 1 and evaluating the term φ_i using Eqn. (3).

The new particle position is given as:

$$x_{i+1} = x_i + r_i \cos \omega_i \qquad (4)$$

$$y_{i+1} = y_i + r_i \cos \omega_i \qquad (5)$$

The walk is regarded as floating as the length depends on the closeness of the current position of the particle to the boundary. The walk is terminated whenever an absorbing boundary is encountered, and the boundary value of such recorded. When the particle reaches an adiabatic boundary it is reflected back into the domain at a specified reflection distance.

A convective boundary could be represented using a finite difference scheme as

$$T(x,y) = \frac{1}{1+\frac{h\,dr}{k}} T_{dr} + \frac{h\,dr}{1+\frac{h\,dr}{k}} T_\infty \qquad (6)$$

where:

T_{dr} is the temperature at a location 'dr' normal to the boundary
h is the convective heat transfer coefficient of the fluid in contact with the boundary
and T_∞ is the free stream temperature of the fluid in contact

For convective boundary, a random number is first selected. If the selected random number is less or equal to $\frac{h\,dr}{1+\frac{h\,dr}{k}}$, then the boundary is treated as being absorbing and a value of T_∞ scored otherwise it is reflected back to a position 'dr' normal to the boundary.

The average of all the scores is used to estimate the solution at the desired point. A modified representative flow chart of the procedure as obtained from Ogundare [17] is presented in Fig. 1.

2.1.1 Special features in application of floating walk to an arbitrary shaped boundary

When the boundary has arbitrary shape, determination of the minimum distance between the particle and the boundary requires special consideration. A boundary is fully described by set of nodes on it. When a particle commences walk at a solution point, its minimum distance to the boundary is determined by identifying the boundary node (i) closest to it. Other neighbouring boundary nodes, i+1 and i-1 are also identified. The possible locations of the particle relative to the three nodes are as shown in Fig. 2. Then the actual distance to the boundary is determined using Table 1.

2.2 Markov Chain Technique

Sadiku et al. [13] discussed the application of Markov chain for analysis of steady state heat conduction.

A given domain was divided into grids and numbered. For any point in the domain the finite difference representation of the steady state heat conduction equation with no heat source is given as:

$$T_{i,j-1} + T_{i,j+1} - 4\,T_{i,j} + T_{i-1,j} + T_{i+1,j} = 0 \qquad (2)$$

Based on eqn.(2), nodes with known temperatures in the domain were first numbered and regarded as absorbing nodes while other nodes are not.

Table 1. Particle distance determination guide

Case	Particle location	Minimum distance
1	if both θ_1 and θ_2 are acute angles	$r_{min} = r \sin \theta_1 (\theta_1 < \theta_2)$ $r_{min} = r \sin \theta_2 (\theta_2 < \theta_1)$
2	if θ_1 and θ_2 are equal or are both obtuse angles	$r_{min} = r$
3	if θ_1 is acute and θ_2 is obtuse	$r_{min} = r \sin \theta_1$
4	if θ_2 is acute and θ_1 is obtuse	$r_{min} = r \sin \theta_2$

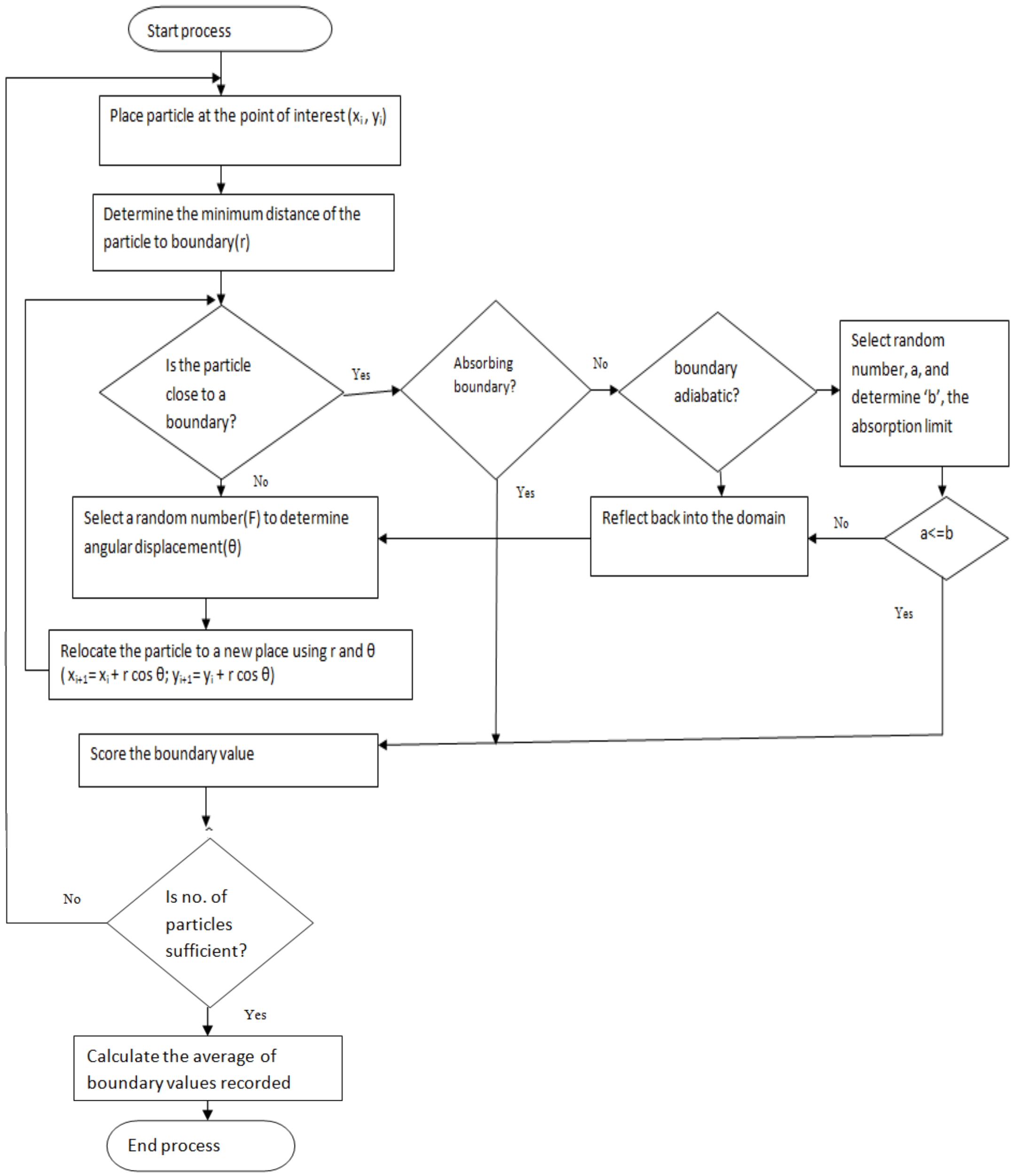

Fig. 1. Flow chart for floating walk technique

Note: $b=\frac{h\,dr}{1+\frac{hdr}{k}}$ *where h is the convective heat transfer coefficient, dr, the reflection distance and k = thermal*

A transition probability Pij which was defined as the probability that a random-walking particle at node 'i' moves to node 'j' was obtained. The probability matrix P formed was recast as

$$P = \begin{bmatrix} I & 0 \\ R & Q \end{bmatrix}$$

Where *R* is the probabilities of moving from non-absorbing nodes to absorbing nodes:

Q- The probability of moving from non absorbing node to another.
I- is an identity matrix
O- Is a null matrix

And Q_{ij} =

$$\begin{cases} 0.25\ if\ i\ is\ directly\ connected\ to\ j \\ 0\ if\ i=j\ or\ i\ is\ not\ directly\ connected\ to\ j \end{cases}$$

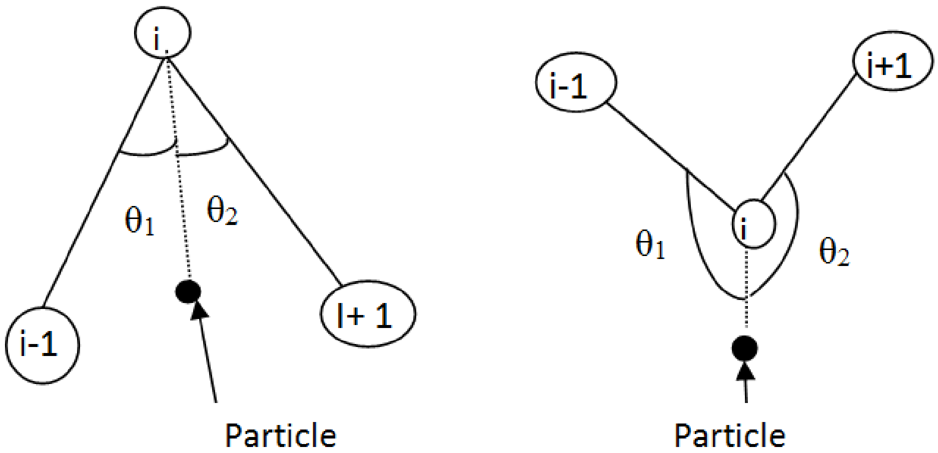

Fig. 2. Possible particle positions close to the boundary
**Note: Only cases when the two subtending angles are either acute or obtuse are shown*

The absorbing probability matrix B was defined as

$$B = NR \qquad (3)$$

With N = $(I - Q)$

$$\text{and } T_f = B\,T_p \qquad (4)$$

T_f = nodal values at non-absorbing nodes
T_p= nodal values at absorbing nodes

The unknown variables T_f were solved using Eqn.(4).

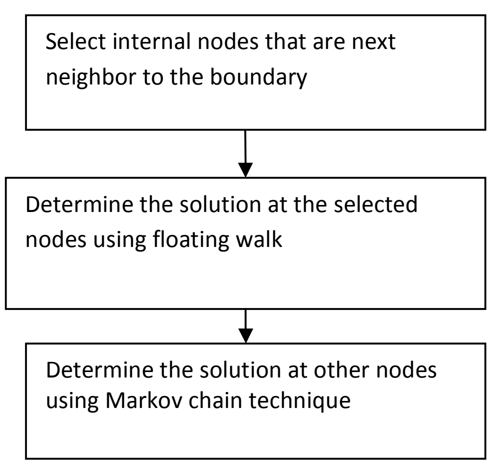

Fig. 3. Implementation flow chart for the hybrid floating walk markov chain technique

2.3 Hybrid Floating Walk and Markov Chain (HFWMC) Techniques

In this technique, internal nodes that are very close to the boundary are first estimated using the floating walk technique. They are then regarded as absorbing nodes when the markov chain technique is subsequently applied. Implementation flow chart for the technique is shown in Fig. 3.

3. APPLICATION OF HFWMC TECHNIQUE TO SOME SELECTED CASES

The developed hybrid floating walk markov chain technique was applied to some selected cases which are hereby discussed.

3.1 Heat Conduction Analysis in a Rectangular Slab with Convective and Adiabatic Boundaries

A rectangular slab with convective and adiabatic boundaries is shown in Fig. 4.

The temperatures at nodes 1, 2, 3, 4, 5, 6, 10, 11, 15, 16, 17, 18 and 19 were first obtained using a floating walk technique. There are then six remaining nodes whose temperatures are evaluated using Markov Chain technique. Nodes with temperatures already evaluated using the floating walk technique are regarded as absorbing nodes. The probability matrix is constructed as:

$$T_1 \quad T_2 \quad T_3 \quad T_4 \quad T_5 \quad T_6 \quad T_{10} \quad T_{11} \quad T_{15} \quad T_{16} \quad T_{17} \quad T_{18} \quad T_{19} \quad T_7 \quad T_8 \quad T_9 \quad T_{12} \quad T_{13} \quad T_{14}$$

$$
P=\begin{matrix} T_1\\ T_2\\ T_3\\ T_4\\ T_5\\ T_6\\ T_{10}\\ T_{11}\\ T_{15}\\ T_{16}\\ T_{17}\\ T_{18}\\ T_{19}\\ T_7\\ T_8\\ T_9\\ T_{12}\\ T_{13}\\ T_{14} \end{matrix}
\begin{bmatrix}
1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0\\
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0&0&0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0\\
0&\frac{1}{4}&0&0&0&\frac{1}{4}&0&0&0&0&0&0&0&0&\frac{1}{4}&0&\frac{1}{4}&0&0\\
0&0&\frac{1}{4}&0&0&0&0&0&0&0&0&0&0&\frac{1}{4}&0&\frac{1}{4}&0&\frac{1}{4}&0\\
0&0&0&\frac{1}{4}&0&0&\frac{1}{4}&0&0&0&0&0&0&0&\frac{1}{4}&0&0&0&\frac{1}{4}\\
0&0&0&0&0&0&0&\frac{1}{4}&0&0&\frac{1}{4}&0&0&\frac{1}{4}&0&0&0&\frac{1}{4}&0\\
0&0&0&0&0&0&0&0&0&0&0&\frac{1}{4}&0&0&\frac{1}{4}&0&\frac{1}{4}&0&\frac{1}{4}\\
0&0&0&0&0&0&0&0&\frac{1}{4}&0&0&0&\frac{1}{4}&0&0&\frac{1}{4}&0&\frac{1}{4}&0
\end{bmatrix}
$$

R- matrix width Q-matrix width

Then the matrices *Q* and *R* are extracted thus:

$$
Q=\begin{bmatrix}
0&\frac{1}{4}&0&0&0&\frac{1}{4}&0&0&0&0&0&0&0\\
0&0&\frac{1}{4}&0&0&0&0&0&0&0&0&0&0\\
0&0&0&\frac{1}{4}&0&0&\frac{1}{4}&0&0&0&0&0&0\\
0&0&0&0&0&0&\frac{1}{4}&0&0&\frac{1}{4}&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&\frac{1}{4}&0\\
0&0&0&0&0&0&0&0&\frac{1}{4}&0&0&0&\frac{1}{4}
\end{bmatrix}
\quad \text{and} \quad
R=\begin{bmatrix}
0&\frac{1}{4}&0&\frac{1}{4}&0&0\\
\frac{1}{4}&0&\frac{1}{4}&0&\frac{1}{4}&0\\
0&\frac{1}{4}&0&0&0&\frac{1}{4}\\
\frac{1}{4}&0&0&0&\frac{1}{4}&0\\
0&\frac{1}{4}&0&\frac{1}{4}&0&\frac{1}{4}\\
0&0&\frac{1}{4}&0&\frac{1}{4}&0
\end{bmatrix}
$$

Matrices *N* and *B* are constructed using the relations discussed in Section 2.2. The nodal values T_7, T_8, $T_{9,}$ $T_{12,}$ T_{13} *and* T_{14} are solved using Eqn. (4) with

$$
Tp=\begin{bmatrix} T_1\\ T_2\\ T_3\\ T_4\\ T_5\\ T_6\\ T_{10}\\ T_{11}\\ T_{15}\\ T_{16}\\ T_{17}\\ T_{18}\\ T_{19} \end{bmatrix}
$$

The temperature estimates using finite difference technique as well as HFWMC (numbered from left to right and top to bottom, 5 x 6 matrix) are presented in Tables 2 and 3 respectively. Both the finite difference and hybrid HFWMC solutions were subjected to statistical analysis using ANOVA (α=0.05). ANOVA yielded a probability value of 0.673 implying that the results from the two techniques are not significantly different. Computer execution time with Pentium CPU B 850, processing speed of 2.1 GHz and 4 GB RAM using finite difference was 3.74 seconds while that of the HFWMC was 30.2 seconds. The computer execution time using only floating walk technique was 99.2 seconds.

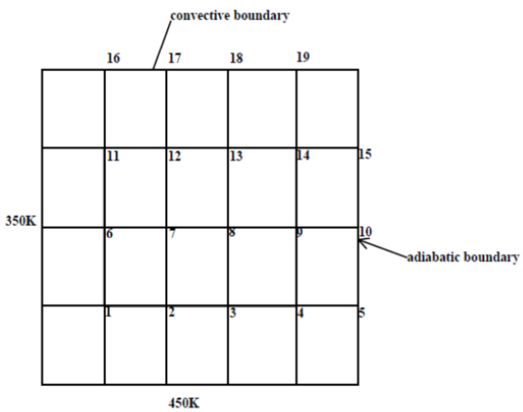

Fig. 4. Rectangular slab with mixed boundaries

3.2 Heat Conduction Analysis in an Arbitrary Shaped Slab with Isothermal Boundaries

Fig. 5 is an arbitrary slab with isothermal boundary conditions. As in the previous case, the temperature at neighbouring nodes were first obtained using floating walk technique before the Markov chain technique was applied.

A plot of obtained temperature values using only floating walk technique and HFWMC technique for all solution points is shown in Fig. 6. As could be observed, values from the two techniques were very close. Statistical analysis using ANOVA(α =0.05) yielded a probability value of 0.978 further confirming that the two techniques yielded almost the same results.

3.3 Heat Conduction Analysis in an Arbitrary Shaped Slab with Mixed Boundaries

Both floating walk and hybrid floating walk markov chain techniques were used to study the heat conduction in an arbitrary shaped slab with mixed boundaries (see Fig. 7). Fig. 7 is a representative of a cut-away section of a turbine blade with internal holes for cooling. The results at all solution points are presented in Tables 4 and 5. Comparison of the values on cell to cell basis showed that they are mostly the same. Statistical analysis using ANOVA(α =0.05) yielded probability value of 0.884 implying that the results from the two techniques are not significantly different.

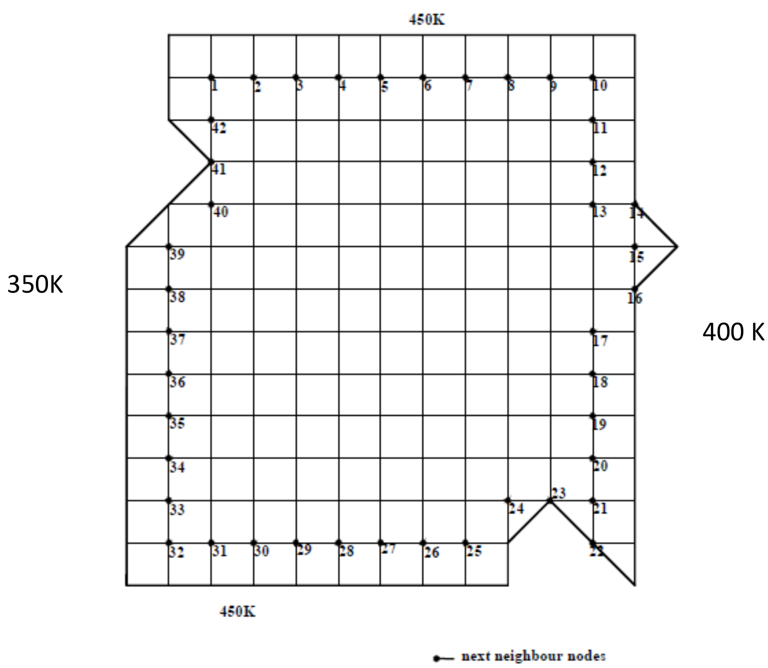

Fig. 5. Arbitrary shaped slab with selected next neighbour nodes

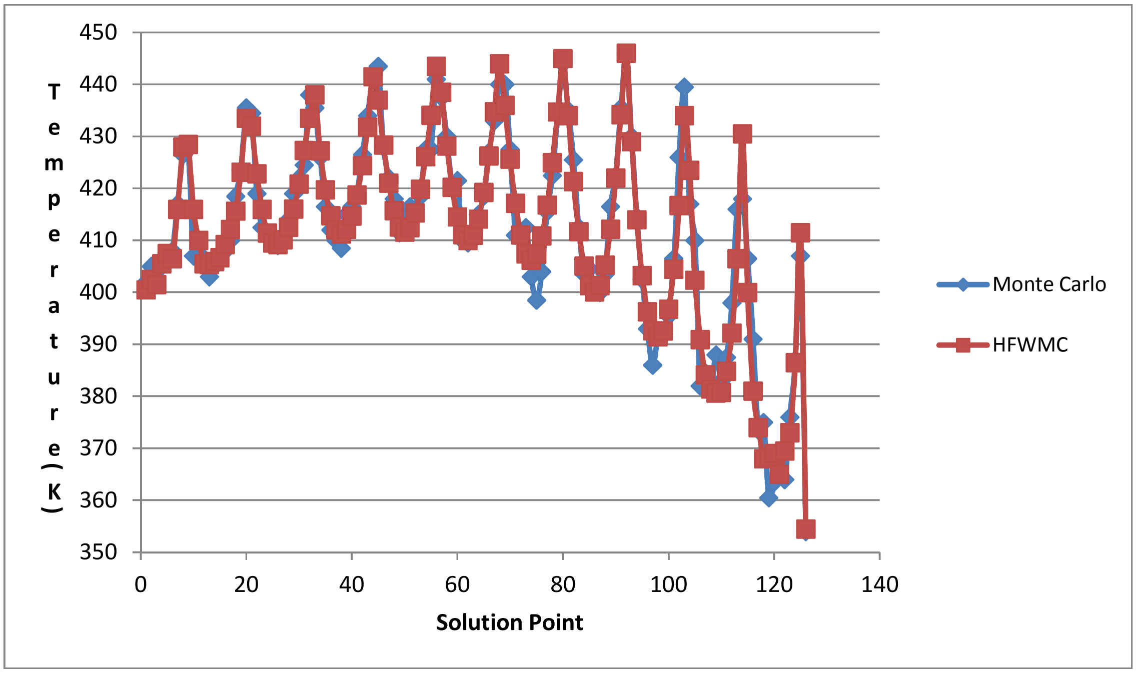

Fig. 6. Plot of temperature for various solution points using floating walk and HFWMC techniques for arbitrary slab with isothermal boundaries

Table 2. Temperature distribution in the rectangular slab using finite difference technique

363.8	377.6(16)	393.6(17)	405.6(18)	412.6(19)	414.9
350.0	376.5(11)	395.5(12)	408.1(13)	415.0(14)	417.2(15)
350.0	382.9(6)	404.0(7)	416.2(8)	422.2(9)	423.8(10)
350.0	401.1(1)	421.4(2)	430.4(3)	434.0(4)	433.4(5)
400.0	450.0	450.0	450.0	450.0	441.7

** the corresponding nodes in Fig. 4 are in brackets*

Table 3. Temperature distribution in the rectangular slab using hybrid floating walk markov chain technique

372.5	395.0(16)	409.0(17)	415.0(18)	412.0(19)	415.5
350.0	380.0(11)	402.6(12)	413.5(13)	417.6(14)	419.0
350.0	386.0(6)	407.8(7)	418.8(8)	425.8(9)	429.0
350.0	403.0(1)	424.0(2)	428.0(3)	438.0(4)	434.0
400.0	450.0	450.0	450.0	450.0	442.0

***** *the corresponding nodes in Fig. 4 are in brackets*

Table 4. Temperature values at grid points using floating walk technique for arbitrary slab with mixed boundaries

435.6	436.2	441.6	442.8	442.2			
434.4	434.4	436.2	445.2	447.6			
432.6	433.8	435.6	439.8	444.6			
	425.4	426.6	429.6	437.4	445.8		
		424.2	424.8	431.4	436.2	441	438.6
			424.2	429.6	431.4	435.6	431.4
			421.8	427.8	431.4	433.8	433.2
			423	428.4	433.2	434.4	433.8

Table 5. Temperature values at grid points using HFWMC technique for arbitrary slab with mixed boundaries

436.2	437.7	440.1	444.6	445.2			
432.0	434.8	437.4	440.8	444.3			
434.7	432.1	433.9	436.9	439.8			
	425.1	429.1	433.0	437.4	443.4		
		424.5	428.6	433.4	437.7	440.1	441.6
			423.6	429.8	433.8	435.9	436.2
			424.5	428.2	432.0	433.4	432.9
			425.1	426.6	432.6	432.9	432.9

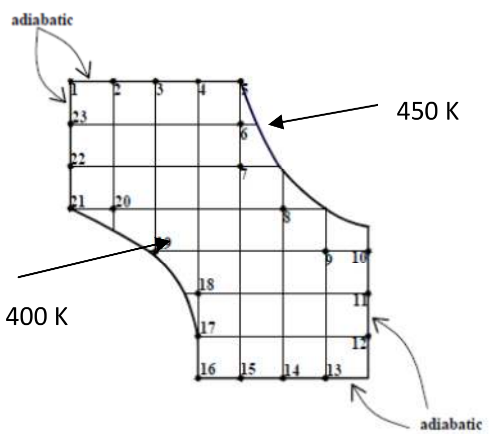

Fig. 7. Grid section of a arbitrary slab with mixed boundaries arbitrary slab with mixed boundaries

4. CONCLUSION

An hybrid technique for numerical simulation of heat conduction has been developed. The developed technique has only been able to handle steady state 2-dimensional heat conduction problems. A faster execution time using the developed technique for heat conduction analysis has been established as compared with the floating walk technique. The technique gave good results when compared with finite difference solution and suitably handled irregular shaped body. The computational time is however longer as compared with the finite difference technique. However in view of the general difficulty in applying finite difference technique for analysis of irregular boundary, the developed technique may be well suited to handle such instances.

COMPETING INTERESTS

Author has declared that no competing interests exist.

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