Mathematical Model of a Closed Loop Pitch Controlled Turbine in a Wind Energy Conversion System

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Author's contribution

The sole author designed, analyzed and interpreted and prepared the manuscript.

Original Research Article

Abstract

The closed loop pitch control relies on the variation in the power captured by the turbine as the pitch angle of the blades is changed in line with most companies' challenge of maintaining a steady and optimal power supply. The control involves the pitch angle, wind power, reference power, wind speed and the angular velocity of the turbine. This is done through modelling and simulation undertaken and implemented in MATLAB/SIMULINK simulation environment to optimize energy capture. In conclusion, pitch function gives full control over the mechanical power for the variable speed wind turbines.

1 Introduction

Wind energy model system can be subdivided into the following components: (i) model of the wind, (ii) turbine model, (iii) shaft and gearbox model, (iv) generator model and (v) control system model. There are two types of design models for wind turbines which are classified on the basis of their axis in which the turbines rotate: Horizontal Axis Wind Turbine (HAWT) and Vertical Axis Wind Turbine (VAWT).The VAWT is also called Darrieus rotor named after its inventor [1]. Wind turbines operate in two modes

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namely constant and variable speed. For a constant speed turbine, it produces less energy at low wind speeds than does a variable wind speed turbine which is designed to operate at a rotor speed proportional to the wind speed below its rated wind speed [2] which its units are usually equipped with a blade pitching system [3].

2 Mathematical Model of Drive Train

The most important part of the drive train is the modelling of the turbine itself as the other parts can be analyzed with common methods. Aerodynamic torque of the turbine is a nonlinear function with respect to the tip speed ratio and the pitch angle. This relation $C_T(v_t, w_t, \beta)$ may be modelled with splines or with look-up tables. The $c_T - \gamma$ curve is linearized with respect to wind speed, rotational speed, and pitch angle (in the pitch controlled turbine) at a given operating point. To analyze the turbine with linear methods, the non-linear torque function in (2.1) can be linearized by taking into account only the first terms of the Taylor series [4].

$$\Delta T_a = \theta \Delta v_t + \gamma \Delta w_t + k \Delta \beta \tag{2.1}$$

$$\theta = \left.\frac{\partial T_a}{\partial v_t}\right|_{O.P} = C_o w_t \left(2 C_{T,0} - \gamma_0 \frac{\partial C_T}{\partial v_t}\right)\Bigg|_{O.P} \tag{2.2}$$

$$\gamma = \left.\frac{\partial T_a}{\partial w_t}\right|_{O.P} = C_0 R w_0 \left.\frac{\partial C_T}{\partial w_t}\right|_{O.P} \tag{2.3}$$

$$k = \left.\frac{\partial T_a}{\partial \beta}\right|_{O.P} = C_0 \left.\frac{\partial C_T}{\partial \beta}\right|_{O.P} \tag{2.4}$$

$$C_0 = \frac{\rho}{2} \pi R^3 \tag{2.5}$$

$$\gamma = \frac{w_{t,0} R}{v_{t,0}} \tag{2.6}$$

With linearization around an operating point, the product of the derivative of aerodynamic torque and wind speed may be regarded as an external disturbance. If the pitch angle control is adopted as a means of control, the linearized coefficient k must be calculated. The dynamics of the pitch actuator may be modelled with the first order dynamics. The derivatives term $\frac{\partial T_a}{\partial \beta}$ can be obtained from blade design calculations or by identification from tests.

2.1 Pitch control system design

Pitch control means that the blades can pivot upon their own longitudinal axis. The pitch control used for speed control, optimization of power production and to start and step the turbine. The control system structure used to generate the pitch angle reference is given in Fig. 2. The pitch controller consists of two

paths a nonlinear feed forward path, which generates β_0 and a linear feedback path, which generates $\Delta\beta$, where β_0 the original pitch angle and $\Delta\beta$ is the change for the pitch angle. The P-I controller for the system is designed using the Zeigler Nicholas rules for tuning PID controllers as shown in Fig. 1. The tip speed ratio is obtained from both quantities. For each wind speed, the potential wind power in front of the converter - P_{wind} - has to be calculated. Then the power coefficient c_p is equal to;

$$C_p(\gamma, \beta) = \frac{P_{TurbM}}{P_{Wind}} = \frac{P_{TurbM}}{0.5\rho A V^3}$$ (2.1.1)
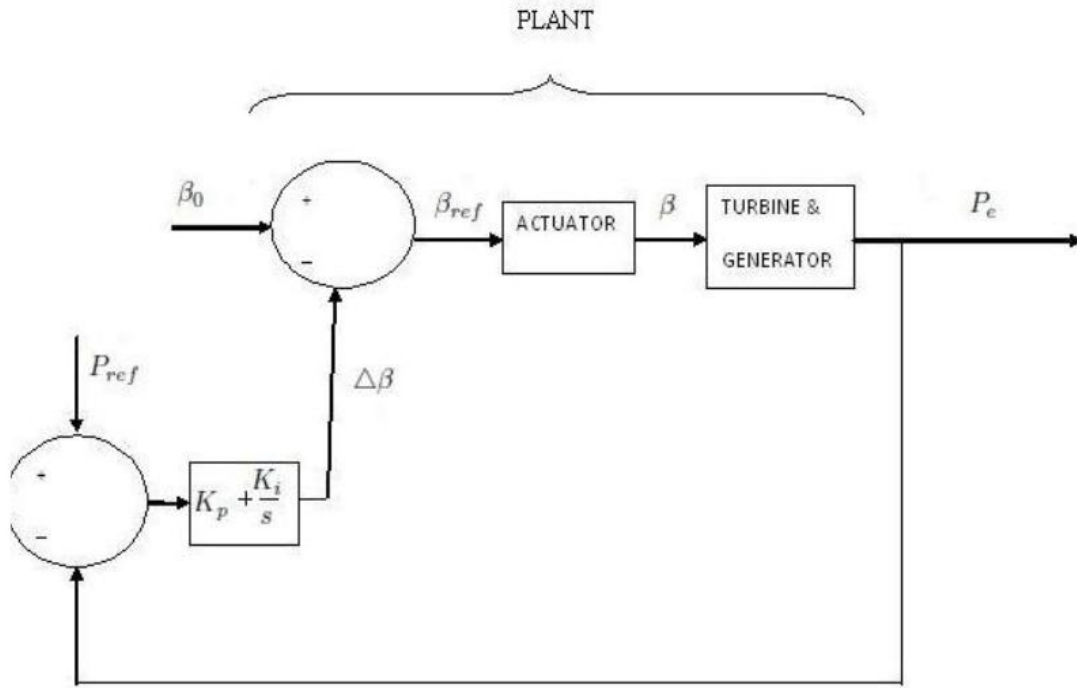

Fig. 1. Plant and controller definition [5]

This is the basic control system, which is applied at normal operation of standard wind turbines. For the modelling of large offshore wind farms the standard model has to be extended. Namely, an additional function will be added which can reduce the produced energy based on an external set point signal. When the signal is sent the standard control module is disabled and instead of the β_{ref} value - the signal β_{ORDR} which corresponds to power reduction will be set as new reference value. The power is reduced as long as the order signal is given. When the order signal is disabled the standard control is restarted again. Another option is the modeling of the safety stop function. This function is activated when the safety stop order is given. Then the standard control module is disabled again and $\beta_{max} = 90^0$ is given as a reference value and the wind turbine will be turned. In the next step, the assigned reference value is compared with the actual pitch angle. The resulting signal is given to the module which represents the pitch servo system. In order to get a realistic response the servo system is represented with a time constant T_{servo}. Additionally, the limitations of the pitch angle and its gradient are implemented. The assumed limit for the pitch angle gradient is equal to $10^0/s$ [6]. The dynamic of actuators and blades can be represented with first or second order system [7].
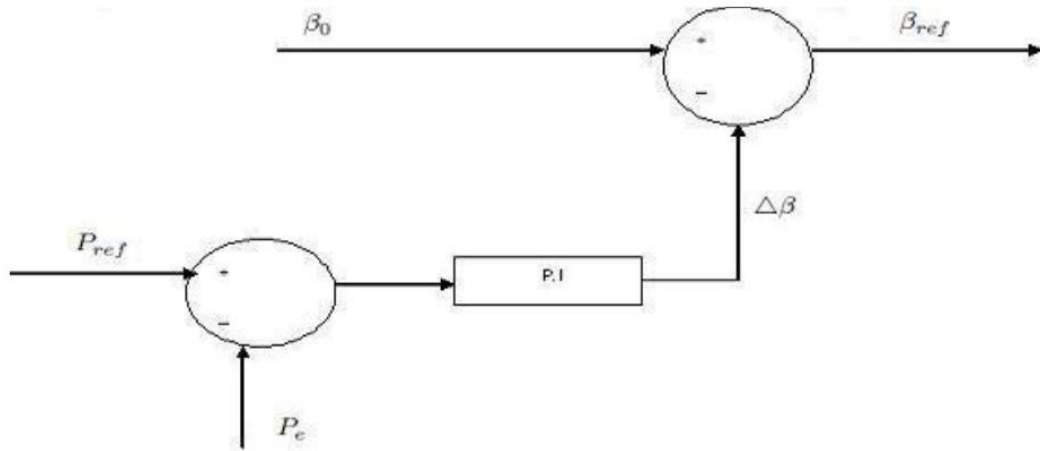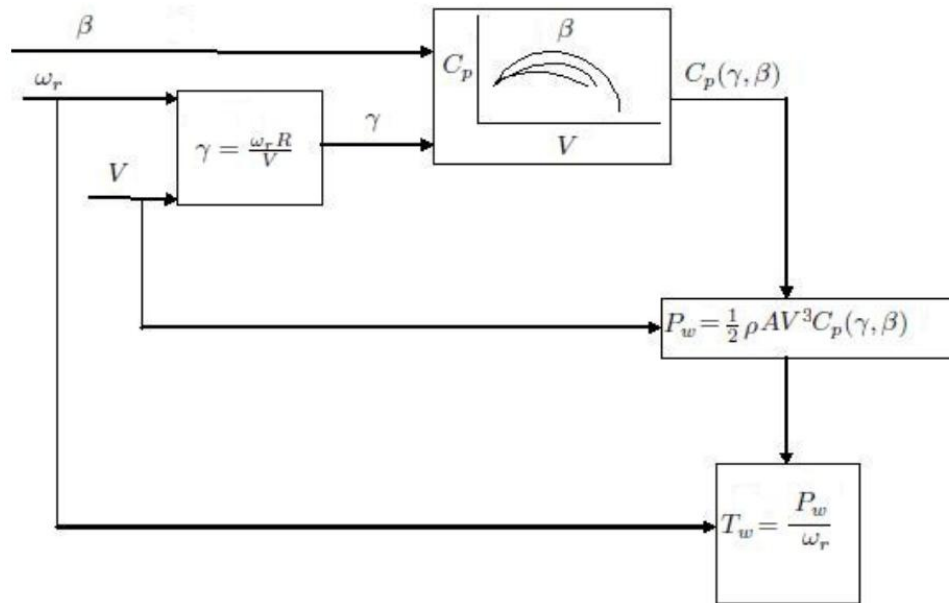
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Fig. 2. Pitch angle reference generator [8]

Fig. 3. Block diagram of the wind turbine [7]

2.2 Closed loop pitch control system model

Some of the main objectives of a wind turbine control system as outlined by [9] are Energy capture and Power quality. The control techniques used in wind turbines are pitch control, yaw control and stall control [10]. The output power or torque of a wind turbine is determined by several factors. Among them are (i) angular velocity of turbine, (ii) rotor blade tilt, (iii)pitch angle of the rotor blades (iv) size and shape of turbine, (v) area of turbine, (vi) rotor geometry whether it is a HAWT or a VAWT, (vii) and wind speed. A relationship between the output power and the various variables constitute the mathematical model of the wind turbine. A mathematical model of wind turbine is essential in the understanding of the behavior of the wind turbine over its region of operation and also modelling enables control of wind turbines performance.
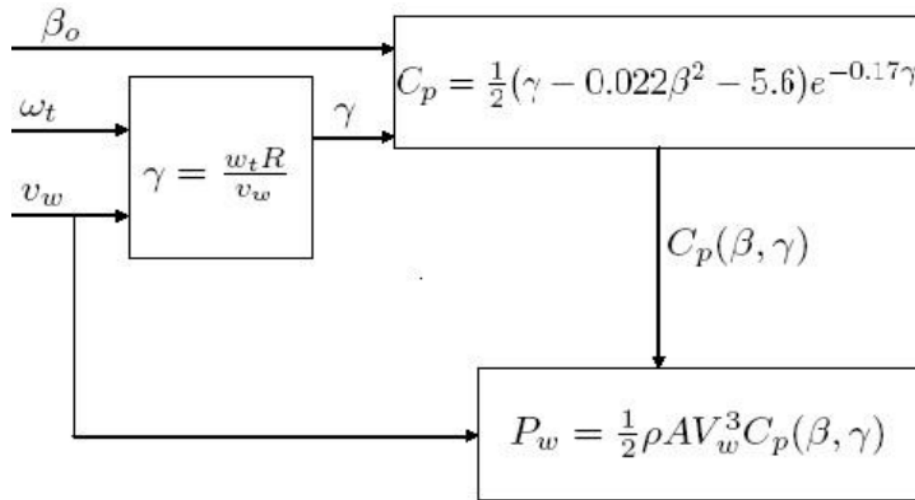
Fig. 4. Control system model

3 Mathematical Formulation of Pitch Control System Model

The kinetic energy in air of mass m moving with speed V is given by;

$$kineticeEnergy = \frac{1}{2}mV^2 \tag{3.1}$$

The power in moving air is the flow rate of kinetic energy per second. Therefore:

$$P = \frac{1}{2}(\rho A V)V^2 = \frac{1}{2}\rho A V \tag{3.2}$$

The actual power extracted by the rotor blades is the difference between the upstream and the downstream wind powers. That is, using Equation (3.2):

$$P_0 = \frac{1}{2}[massflowrate][V^2 - V_o^2] \tag{3.3}$$

Where;

P_0 -mechanical power extracted by the rotor

V -upstream wind velocity at the entrance of the rotor blades

V_0 -downstream wind velocity at the exit of the rotor blades

Mass ow rate of air through the rotating blades is derived by multiplying the density with the average velocity .i.e.

$$massflowrate = \frac{1}{2}\rho A \frac{V + V_0}{2} \tag{3.4}$$

Replacing equation 3.4 into equation 3.3 gives;

$$P_0 = \frac{1}{2}[\rho A \frac{V + V_0}{2}](V^2 - V_0^2) \tag{3.5}$$

The above expression can be algebraically rearranged:

$$P_0 = \frac{1}{2}\rho A V^3 \frac{(1 + \frac{V_0}{V})[1 - \left(\frac{V_0}{V}\right)^2]}{2} \tag{3.6}$$

The power extracted by the blades is expressed as a fraction of the upstream wind power as follows:

$$P_0 = \frac{1}{2}\rho A V^3 C_p \tag{3.7}$$

Where;

$$C_p = \frac{(1 + \frac{V_0}{V})[1 - \left(\frac{V_0}{V}\right)^2]}{2} \tag{3.8}$$

Equation 3.8 shows that the turbine captures only a fraction of this power.

The power captured by the turbine (P_0) can be expressed as [11],

$$P_0 = P_w \times C_p \tag{3.9}$$

where C_p is a fraction called the power coefficient. The power coefficient represents a fraction of the power in the wind captured by the turbine and has a theoretical maximum of 0.55 [12]. The power coefficient can be expressed by a typical empirical formula as

$$C_p = \frac{1}{2}\left(\gamma - 0.022 \beta^2 - 5.6\right)e^{-0.17\gamma} \tag{3.10}$$

where β is the pitch angle of the blade in degrees and is the tip speed ratio of the turbine, defined as

$$\gamma = \frac{v_w (mph)}{w_b (rads^{-1})} \tag{3.11}$$

where, w_b - turbine angular speed.

One concept that is fundamental to the control dynamics is that the speed change is relatively slow because of the large inertia involved. Pitch control can be better used to regulate power flow especially when near the high speed limit. The generator output can be controlled to follow the commanded power. From equation (3.9) it can be shown that;

$$C_p = \frac{P_0}{P_w} \tag{3.12}$$

where $P_m = P_{t \arg et}$

TSR is the ratio between the linear speeds of the tip of the blade with respect to the wind speed. The power coefficient C_p varies with the tip-speed ratio, and TSR is given by the equation;

$$\gamma = \frac{w_t R}{V} \tag{3.13}$$

Hence,

$$w_t = \frac{\gamma_{opt}}{R} V \tag{3.14}$$

From equation (3.7), the power production from the wind turbine can be maximized if the system is operated at maximum C_p. As the wind speed changes, the rotor speed should be adjusted to follow the change. This is possible with a variable-speed wind turbine. Wind speed cannot be reliably measured thus to avoid using the wind speed, the equation to compute the target power can be rewritten by substituting the wind speed V and the C_p in the power extracted from the wind given by;

$$P_w = \frac{1}{2} \rho A V_w^3 C_p (\beta, \gamma) \tag{3.15}$$

The target power $P_{t\,arg\,et}$ can then be written as;

$$P_{t\,arg\,et} = \frac{1}{2} \rho A C_{p-t\,arg\,et} \left[\frac{R}{\gamma_{t\,arg\,et}} \right]^3 w_t^3 \tag{3.16}$$

It can be seen that the $P_{t\,arg\,et}$ is proportional to the cube of the rotor speed. To prevent rotor speed from becoming too high, the extracted power from incoming wind must be limited. This can be done by reducing the coefficient of performance of the turbine (the C_p value).

Since $C_{p-t\,arg\,et} = C_{p-\max} = \text{constant}$, in this region, from (3.16):

$$P_w = KV^3 \tag{3.17}$$

where;

$$K = \frac{1}{2} \rho A C_{p-\max} (\beta, \gamma) = \text{Constant} \tag{3.18}$$

The wind speed is varied, turbine speed is maintained at rated speed $w_t = w_{t-rated}$ and corresponding γ is calculated using (3.13). The power output is maintained at rated power $\left(P_{rated} = P_{t\,arg\,et} \right)$. The C_p corresponding to rated power is calculated using:

$$C_p = \frac{P_{target}}{0.5\rho A V_w}$$

(3.19)

Therefore,

$$\beta = \sqrt{\left(\frac{1}{0.022}\left(\gamma - 5.6 - \frac{2P_{target}e^{-0.17\gamma}}{P_w}\right)\right)}$$

(3.20)

4 Results and Discussion

Pitch angle control of the wind energy conversion system were simulated using MATLAB/SIMULINK tool to test the control strategy and evaluate the performance of the system. From the figure, the power and rotor speed of the wind energy conversion system was controlled at different pitch angles which increases with the growth of wind speed and can be regulated to limit the power output to its rating when the wind speed increases. The C_p is a non-linear function of γ, which is determined by the pitch angle. Since the C_p is a function of γ, for each wind velocity, there is a turbine speed that gives a maximum output power as shown by the peak power points for various values of β and γ. When γ has a drop, pitch angle control is actuated to rise, so as to lower the extracted power which will keep the γ value showing that the wind speed is proportionate to rotational speed of the turbine. When γ varies between 10-15, the pitch response is rapid and when it varies between 16-25, the pitch response slows down. In normal operation, closed loop pitch angle control with rotational speeds is expected between $\beta_n = 0^0 - 10^0$. Here it's clear that there is a reduction of C_p from 0.402 to about 0.201. The simulation also shows that the maximum C_p is achieved at a particular γ, which is specific to the aerodynamic design of a given turbine.
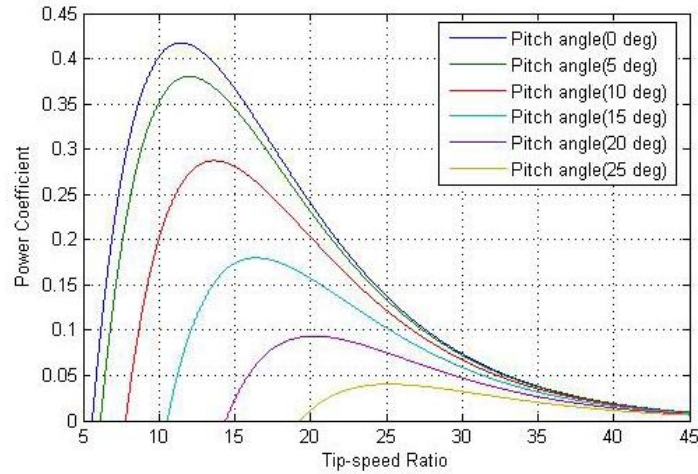

Fig. 5. Power Coefficient vs. tip speed ratio for various values of pitch angles

5 Conclusion

In conclusion, the simulation shows that the pitch function gives full control over the mechanical power for the variable speed wind turbines. Wind turbine can be operated at its optimum energy capture while

minimizing the load on the wind turbine for a wide range of wind speeds and it is operated at high power coefficient values most of the time and these values are manipulated by changing the pitch angle. Altering the pitch angle means slightly rotating the turbine blades along the axis facilitated by either hydraulic or electric drives. This paper proposes the closed loop pitch-angle control for limiting the mechanical power if the turbine over speeds and enable the control of the WECS output real power, from a low value up to the maximum power corresponding to the prevailing wind speed.

Competing Interests

Author has declared that no competing interests exist.

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