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The Combined Effects of Electro-osomotic and Magnetohydrodynamic with Viscosity and Thermal Conductivity in Reactive Fluid Flow

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Authors' contributions

This work was carried out in collaboration between both authors. Both authors read and approved the final manuscript.

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Abstract

The study of combined effects of electro-osmotic and magnetohydrodynamic when viscosity and thermal conductivity of the reactive fluid flow is assumed to vary exponentially with temperature is examined. The governing equations of the flow were non-dimensionalized using suitable variables. The Galerkin weighted residue method was adopted to solve both the momentum and energy equations in the steady state for a constant viscosity and thermal conductivity. It was observed that velocity profile increases as pressure, electro-osmotic and specific internal energy parameters increases and decreases as viscosity, magnetic and electro-kinetic parameters increases.

Keywords: Electro-osmotic; magnetohydrodynamic; viscosity; thermal conductivity; reactive fluid.

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1 Introduction

Newtonian fluids are fluids that obey Newton's law of viscosity and for which μ has a constant value. More precisely, a fluid is Newtonian only if the tensors that describe the viscous stress and strain rate are related by a constant viscosity tensor that does not depend on the stress state and velocity of the flow. Most common liquids and gases such as water and air can be assumed to be Newtonian for practical calculations under ordinary conditions. Zeta potential is the electrical potential at the shear plane. Application of an electric field along the length of the micro–plates causes an electrical body force to be exerted on the mobile ions in the diffuse layer. Then the ions move under the influence of electrical field and move the liquid by viscous forces. This type of flow is called electro-osmotic flow (EOF) [1,2,3,4].

Magnetohydrodynamic (MHD) studies the magnetic properties of electrically conducting fluids such as Plasmas, liquid metals and saltwater or electrolytes. The fundamental concept behind MHD is that magnetic fields can induce currents in a moving conductive fluid which in turn polarizes the fluid and reciprocally changes the magnetic fluid itself. More so, reactive fluid flow have received increasing attention for studies of contaminant transport ground water quality, waste disposal, acid mine drainage, remediation, mineral deposits, sedimentary just to mention a few [5]. However, a liquid is said to be viscous, if its viscosity is substantially great. Joule heating is the process by which the passage of an electric current through a conductor releases heat. The amount of heat release is proportional to the square of the current such that $Q \propto l^2$. This is caused by interactions between the moving particles that form the current (but not always electrons) and the atomic ions that make up the body of the conductor [6-8].

This fluid has wide applications in many branches of science and engineering of most focus is the thermal behaviours of fluids whose viscosity changes with temperature and the flow is accompanied by a simultaneous transfer of mass, energy and momentum in the system due to reaction occurring between the fluids The ability to adequately describe such system is necessary for the prediction of its thermal stability among others. Hence, Efforts have been devoted to the study of heat transfer and thermal stability of reacting Newtonian fluids that is of extreme importance not to compromise on safety of life and materials during handling and processing of such fluids and for quality control purposes in many manufacturing and processing industries [9]. An improvement in thermal recovery and utilization during the convention flow in any fluids is one of the fundamental thermal integration of such systems that provide a better materials processing, energy conservation and more environmentally being process [10,11].

The possibility of the existence of a considerable resistance to heat transfer between the reacting fluids and system as a result of low conducting fluids or highly conductive vessel wall, resulting in significant temperature gradient, as was reported by Frank – Kamanetskii [12]. In recent time, the mathematical formation of thermally critical system mainly focuses on the determination of the critical regions separating the regions of explosivity and non explosivity of various works on stability of flows was examined by Billingham [13]. Yihao Zhery et al. [14] investigated the kinetic behaviour and hydrodynamics of pressure driven Poiseuille flow. Makinde [15] studied the thermal stability of a reactive third – grade liquid flowing steadily between two parallel plates with symmetrical convective cooling at the walls. Hence the study of electroosmotic, magnetohydrodynamics with variable viscosity and thermal conductivity in reactive flow is significant for practical reasons.

The present study investigate combined electro-osmotic and magnetohydrodynamic with viscosity and thermal conductively in reactive fluid flow and determine the effect of fluid parameters on velocity profile and temperature profile for a steady, constantviscosity and thermal conductivity in a reactive fluid flow using Galerkin weighted residual method.

2 Problem Formulation

An incompressible viscous fluid flow of the combined effects of electro osmotic and magnetohydrodynamics with viscosity and thermal conductivity in a reactive fluid flow between two parallel plates was investigated. The equations governing the motion of the fluid are momentum, energy and electrical potential written as:

$$\rho\left(\frac{\partial u}{\partial t} + V_o \frac{\partial u}{\partial y}\right) = \frac{\partial}{\partial y}\left(\mu \frac{\partial u}{\partial y}\right) + E_X \rho_e - \frac{\partial p}{\partial x} - \beta_0^2 \sigma u \qquad (1)$$

$$P_{c_p}\left(\frac{\partial T}{\partial t} + V_o \frac{\partial T}{\partial y}\right) = \frac{\partial}{\partial y}\left(k \frac{\partial T}{\partial y}\right) + \mu\left(\frac{\partial u}{\partial x}\right)^2 + \beta_0^2 u^2 + QC_0 A \exp\left[\frac{-E}{RT}\right] \qquad (2)$$

$$\frac{d^2\psi}{dy^2} = \frac{\rho_e}{\varepsilon} \qquad (3)$$

The boundary and the initial conditions of the flow are as follows:

$$u(y, 0) = 0,\ T(y, 0) = T_O$$

$$u(-h, t) = 0,\ u(h, t) = 0$$

$$T(-h, t) = T_o,\ T(h, t) = T_o$$

$$\frac{d^2\psi}{d^2 y}(0) = 0,\ \psi(1) = \varsigma$$

where ρ is the density, μ the viscosity, C_p heat capacity with constant pressure, U and V_o velocity components along x and y axis respectively, T the temperature, P is pressure, K the thermal conductivity, x the co-ordinate in the direction of flow, E the activation energy, R the universal gas constant, Q heat released per unit mass during reactions.

σ the electric field conductivity, ψ the electrical potential, ρ_e the net electric charge density, ε the dielectric constant, β_o the magnetic field, C_O the constant pressure gradient, E_X the Electrical field and A is the rate of heat reaction.

A temperature dependent viscosity is assumed to be $\mu = \mu_0 \exp(\alpha[T - T_0])$ and the following non–dimensional variables were introduced.

where t_o, μ_o are references time and viscosity respectively

$$\phi = \frac{u}{v_o},\ \bar{y} = \frac{y}{h},\ \bar{t} = \frac{t}{t_o}, \rho_e = -2\left(z\ell\eta_o \sinh\left(\frac{z\ell\psi^*}{k_b\theta^*}\right)\right) for\ \theta < 1\ (i.e\quad \sin\theta = \theta)$$

$$\Rightarrow \sinh\theta = \theta,\ hence \quad \rho_e = \frac{-2Z^2\ell^2\eta_0\psi^*}{K_b\theta^*},\ k = k_0 \exp\alpha(T - T_0) \qquad (4)$$

where η_o is bulk ionic concentration and z is valence of type –i ions

Substituting (4) into (1) to (3) with the initial and boundary conditions gives;

$$\frac{\partial u}{\partial t}+\frac{\partial u}{\partial y}=\gamma\frac{\partial}{\partial y}\left[\exp\ \lambda\theta\ \frac{\partial u}{\partial y}\right]+N\psi+p-Lu \tag{5}$$

$$\frac{\partial\theta}{\partial t}+a\frac{\partial\theta}{\partial y}=d\ \frac{\partial}{\partial y}\left[\exp\ \lambda\theta\ \frac{\partial\theta}{\partial y}\right]+g\ \exp\ (\lambda\theta)\left[\frac{\partial\theta}{\partial y}\right]^2+Ju^2+F\ *\ \exp\left[\frac{\theta}{1+\varepsilon\theta}\right] \tag{6}$$

$$\frac{d^2\psi}{dy^2}=K^2\psi \tag{7}$$

Where $K=Ze\sqrt{\frac{2\eta_0}{Ek_b\theta}}$, $L=\frac{\beta_0^2\ \sigma\ t_o}{\rho}, P=-\frac{1}{\rho v_o}\frac{\partial\rho}{\partial x}, N=\frac{t_o\ E_X}{v_o\rho}, \gamma=\frac{\mu_0}{\rho h^2}$,

$a=\frac{v_0t_0}{h},\ d=\frac{k_0t_0}{\rho c_p h^2},\ g=\frac{u_0v_0Et_0}{\rho c_p h^2},\ d=\frac{kt_0}{\rho c_p h^2 RT_0^2},\ J=\frac{\beta_0^2 v_0^2 Et_0}{RT_0^2\rho c_\rho}\quad F=\frac{QC_0\ A\exp\left[-E/RT\right]Et_0}{RT_0^2\rho c_\rho}$

and K is the Debye Huckel parameter and 1/K is Debye characteristics thickness of EDL (electric double layer).

The corresponding initial and boundary conditions are:

$u(y,0)=0,\qquad \theta(y,0)=0$

$u(-1,t)=0,\qquad u(1,t)=0$

$\theta(-1,t)=0,\qquad \theta(1,t)=0$

$\frac{d^2\psi}{d^2y}(0)=0,\ \psi(1)=\varsigma$

For a steady case at $\lambda=0$, $\psi(y)=\frac{\varsigma\cos h\ Ky}{\cos h\ K}$, linearizing the exponential activation energy term (i.e. $e^\theta=1+\theta$), equations (5) and (6) become

$$\frac{du}{dy}=\gamma\frac{d^2u}{dy^2}+N\frac{\varsigma\cos h\ Ky}{\cos h\ K}+p-Lu \tag{8}$$

$$\frac{d\theta}{dy}=d\ \frac{d^2\theta}{dy}+g\left[\frac{du}{dy}\right]^2+Ju^2+f\ (1+\theta) \tag{9}$$

corresponding boundary conditions are:

$u(-1)=0,\qquad u(1)=0$

$\theta(-1)=0,\qquad \theta(1)=0$

3 Method of Solution

It is not easy to solve or get an exact analytical solution to nonlinear problem. Therefore semi analytical solution in terms of weighted residual method was employed for easy and accurate approximate techniques for solving such non linear differential equations.

The Galerkin Weighted Residual Method (GWRM) requires inner product, basis of a vector space of functions which is the same as the weight functions. So for GWRM, a weighted residual method uses a finite number of functions $\{\phi_i(x)\}_{i=0}^{n}$, The differential equation of the problem is

$D(U) = L(U(x)) + f(x) = 0$ on $B[U] = [a,b]$, where 'L' is a differential operator and 'f' is a given function. A trial function of U was introduced to solve the problem:

$$\mathrm{U} \approx u(x) = \phi_0(x) + \sum_{j=1}^{n} C_j \phi_j(x)$$

the Residual were defined as:

$$R(x) = D[u(x)] = L[u(x)] + f(x)$$

Thereafter, an arbitrary weight functions $w(x)$ was choose from the basis functions ϕ_j,

then $$< w, R > = \int_a^b \phi_j(x) D[u(x)] dx = \int \phi_j(x) \{D[\phi_0(x) + \sum_{j=1}^{n} C_j \phi_j(x)]\} dx = 0$$

from the concept of inner product and orthogonality.

These are the set of n-order linear equations which must be solve to obtain all the C_j coefficients.

Using GWRM- on the non- homogeneous equations (8) momentum and (9) equilibrium problems, resulted into the Velocity and Temperature profile functions as given below respectively

$$\begin{aligned}
u(y) = {} & \frac{7}{16}\frac{1}{K^5 \cosh K e^K (L^2 + 28L\gamma + 63\gamma^2)}(-2K^5 p \cosh K e^K L - 72K^5 p \cosh K e^K \gamma + 15Ne^{2K}\varsigma K^3 L \\
& - 45Ne^{2K}\varsigma K^3\gamma - 105Ne^{2K}\varsigma K^3 L + 15N\varsigma K^3 L - 270Ne^{2K}\varsigma K^2\gamma - 45N\varsigma K^3\gamma + 270Ne^{2K}\varsigma KL + 105N\varsigma K^2 L \\
& + 945Ne^{2K}\varsigma K\gamma + 270N\varsigma K^2\gamma - 270Ne^{2K}\varsigma L + 270N\varsigma KL - 945Ne^{2K}\varsigma\gamma + 945N\varsigma K\gamma + 270N\varsigma L + 945N\varsigma\gamma)(1 - y^2) \\
& + y^2(1 - y^2)\frac{21}{16}\frac{1}{K^5 \cosh K e^K (L^2 + 28L\gamma + 63\gamma^2)}(2K^5 p \cosh K e^K L + 45K^5 p \cosh K e^K \gamma + 15Ne^{2K}\varsigma K^3 L \\
& - 255Ne^{2K}\varsigma K^2 L + 45N\varsigma K^3 L - 630Ne^{2K}\varsigma K^2\gamma + 105N\varsigma K^3\gamma + 630Ne^{2K}\varsigma KL + 255N\varsigma K^2 L + 1575Ne^{2K} \\
& + 630N\varsigma K^2\gamma - 630Ne^{2K}\varsigma L + 630NK\varsigma L - 1575Ne^{2K}\varsigma\gamma + 1575NK\varsigma\gamma + 630N\varsigma KL + 1575N\varsigma\gamma)
\end{aligned}$$

$$\theta(y) = -\frac{1}{3432}\frac{1}{F^2-28Fd+63d^2}(3432FJC_1^2+208FJC_1C_2-24FJC_2^2-1716FC_1^2g-3432FC_1C_2g$$
$$-52FJC_2^2g-78936JC_1^2d-15600JC_1C_2d-1944JC_2^2d-72072C_1^2dg+3432C_2C_1dg-18408dgC_2^2$$
$$+3003F^2-108108Fd)(1-y^2)+-\frac{1}{1144}\frac{1}{F^2-28Fd+63d^2}(-1144FJC_1^2+1040FJC_1C_2+264FJC_2^2$$
$$+12012FC_1^2g+5720FC_1C_2g+2028FC_2^2g+8008JC_1^2d-1456JC_1C_2d-504JC_2^2d-24024C_1^2dg$$
$$-16016C_2C_1dg-3640dgC_2^2+3003F^2)y^2(1-y^2)$$

Having obtained the constants of the trial functions as above, values where substituted for thermodynamic parameters p(pressure), L (magnetic), γ (viscosity), N (electro osmotic), K (electro-kinetic), ς (specific internal energy) into the velocity profile function and these thermodynamic parameters substituted, values where varied, resulted into Figs. 1-6. The same was done for thermodynamic parameters of the temperature profile function, and resulted into Figs. 7-10.

4 Results and Discussion

In this study, GWRM was employed to find the velocity and temperature profile functions of an incompressible viscous, combined Electroosomoticand MHD with viscosity in a reactive fluid flow between two parallel plates. The solutions were shown graphically. Effect of various fluid thermodynamic parameters is shown in Figs. 1-10. From Fig. 1 increase in pressure parameter produce an increase in the velocity component of the fluid, this is due to the fact that high pressure exerted on fluid property makes its velocity rises. This is also seen from Fig. 4 where increase in electro-osmotic parameter gives high velocity due to the fact that applied potential force across the ends of the wall results into fluid flow and apparently increase the velocity of the fluid flow .In addition, from Fig. 6 increase in specific internal energy also produces high velocity on the constituent molecules of the fluid. This is because of the thermodynamic pressure for a flowing fluid.

Further more from Figs. 2, 3 and 5. It is observed that increase in the various parameters decreases the velocity component of the fluid. Precisely from Fig. 2 applied magnetic field produces a drag in the form of Lorentz force thereby giving rise to decreasing magnitude of the fluid velocity. More so since the viscosity of this present work is constant, low velocity was observed for higher viscosity parameters base on Newton law of viscosity from Fig. 3. Little difference was seen in the electro-kinetic. Fig. 5 depict that electro-kinetic parameter increases from 2, 4 to 6, gives a very small decrease in the velocity profile.

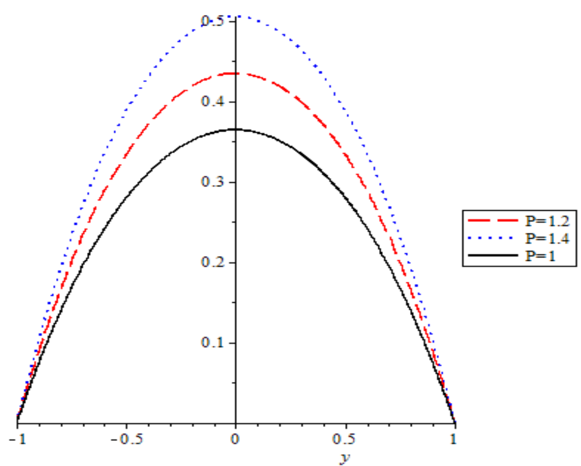

Fig. 1. Velocity profiles for different values of pressure

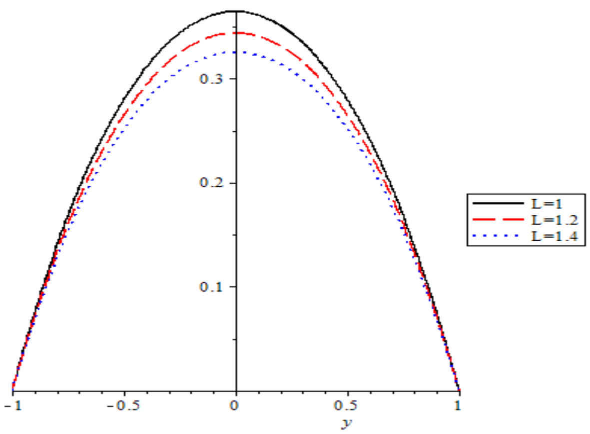

Fig. 2. Velocity profiles for different values of magnetic term

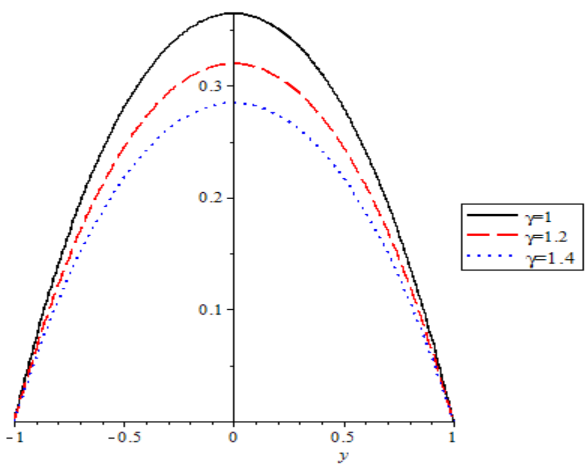

Fig. 3. Velocity profiles for different values of viscosity

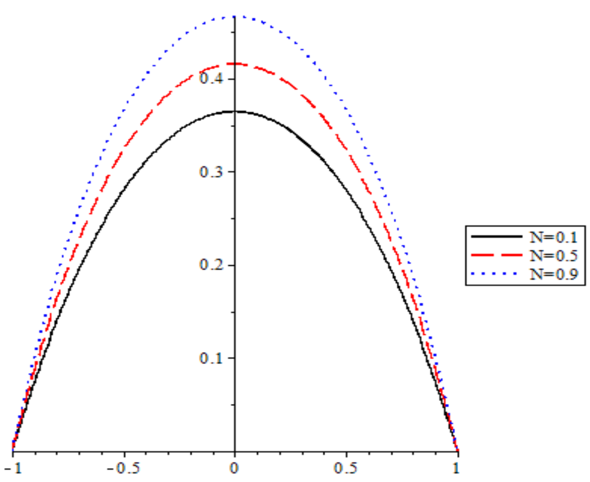

Fig. 4. Velocity profiles for alues of electroosmotics term

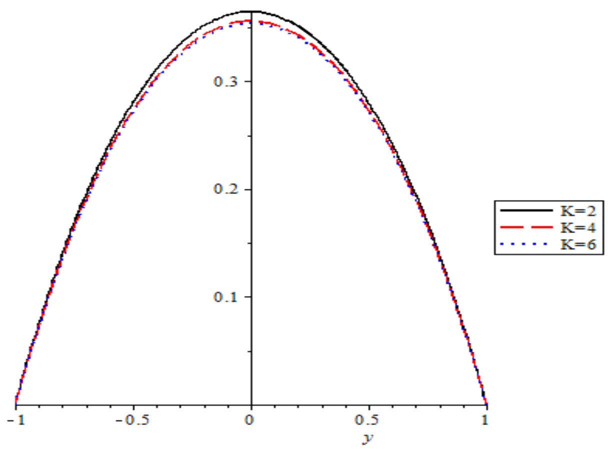

Fig. 5. Velocity profiles for different values of electro-kinetic term

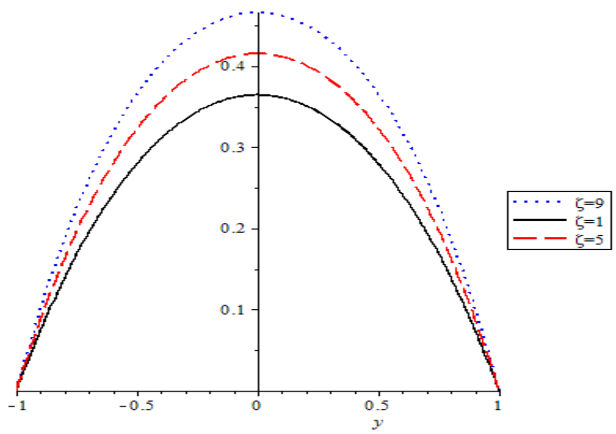

Fig. 6. Velocity profiles for different values of specific internal energy term

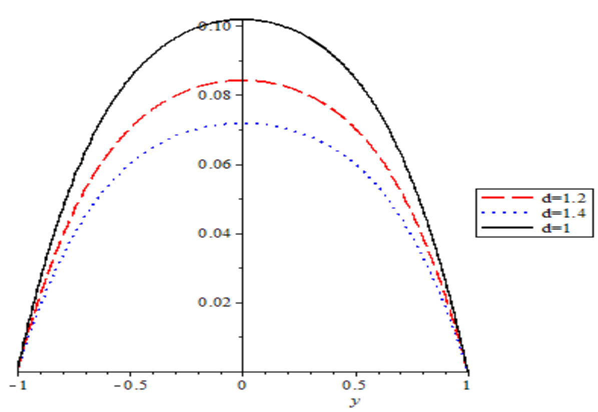

Fig. 7. Temperature profiles for different values of thermal conductivity

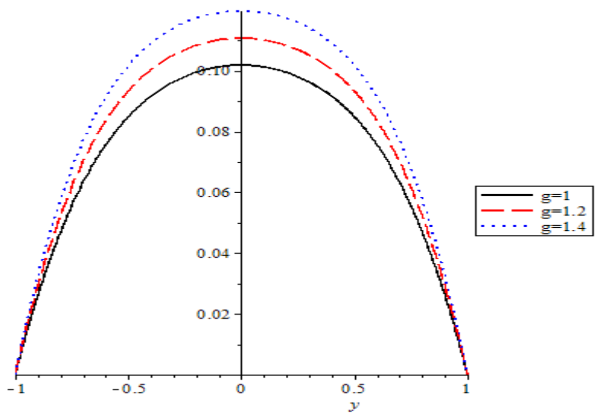

Fig. 8. Temperature profiles for different values of viscosity

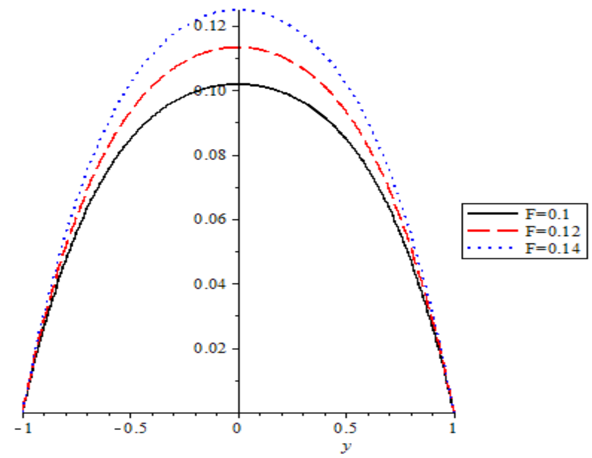

Fig. 9. Temperature profiles for different values of reactive term

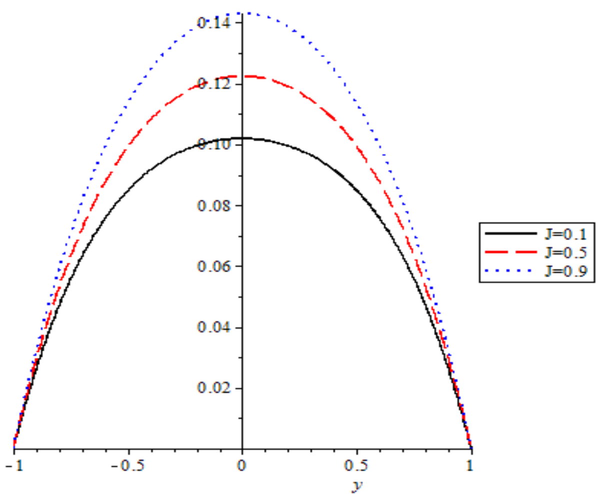

Fig. 10. Temperature profiles for different values of magnetic term

From Figs. 1, 4 and 6, it is seen that the velocity profiles increases as the values of different parameters such as pressure (p), electro-osmotic parameter (N) and specific internal energy (ζ) increases. However, Figs. 2, 3 and 5 show that variation in the values of the parameters viscosity (γ), magnetic (L) and electro-kinetic (K) terms decreases the velocity profiles of each of the parameters.

Furthermore, from Fig. 7, the temperature profile decrease as thermal conductivity (d) parameter increases. more so the temperature profile increases as different parameters of viscosity (g), reactive (f) and magnetic term (g) increases.

5 Conclusion

The combined effect of electro-osmotic, magnetohydrodynamic with viscosity and thermal conductivity shows a direct relationship with velocity profile and temperature profile of a reactive fluid flow. The influence of electro-osmotic and magnetic field on the flow fluid is significant as the parameters retarded the flow while thermal conductivity and viscosity enhances the temperature field due to the thickness in thermal boundary layer as the parameter increases.

Competing Interests

Authors have declared that no competing interests exist.

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