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Applying G-metric Space for Cantor's Intersection and Baire's Category Theorem

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Author's contribution

The sole author designed, analyzed and interpreted and prepared the manuscript.

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Abstract

In this paper, Cantor's intersection theorem and Baire's category theorem are proven by using G-metric spaces.

Keywords: G- metric space; G-open; G-closed; G-closure; G-Cauchy.

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1 Introduction

The concept of G- metric spaces was introduced by Mustafa and Sims [1] in order to extend and generalize the notion of metric space. Many authors have focused on the fixed point results in G- metric spaces (see e.g.[2, 3, 4, 5]).Also, Dhanorkar and Salunke proved fixed point theorems in semicompatibility with an unbounded G-metric spaces [6], and using contractive condition of integral type [7]. In this paper, I am proving Cantor's intersection theorem and Baire's category theorem by using the concept and properties of G-metric spaces.

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2 Preliminary Notes

Definition 2.1 (see[1]). *Let X be a nonempty set and $G : X \times X \times X \to \mathbb{R}^+$ be a function satisfying the following properties:*
(i) $G(x, y, z) = 0$ if $x = y = z$ (Coincidence),
(ii) $G(x, x, y) > 0$, for all $x, y \in X$ with $x \neq y$,
(iii) $G(x, x, z) \leq G(x, y, z)$, for all $x, y, z \in X$ with $z \neq y$,
(iv) $G(x, y, z) = G(\rho\{x, z, y\})$, where ρ is a permutation of x, y, z (Symmetric),
(v) $G(x, y, z) \leq G(x, a, a) + G(a, y, z)$ for all $x, y, z, a \in X$ (Rectangular inequality).

Then the function G is called a generalized metric, or a G-metric on X, and the pair (X, G) is called a G-metric space.

Every G-metric on X defines a metric d_G on X by

$$d_G(x, y) = G(x, y, y) + G(y, x, x) \; for \; all \; x, \; y \in X.$$

The following are examples of G-metric spaces.

Example 2.2 (see[1]). *Let $(\mathbb{R}, d)$ be the usual metric space. Define G_s by*

$$G_s(x, y, z) = d(x, y) + d(y, z) + d(x, z)$$

for all $x, y, z \in \mathbb{R}$. Then it is clear that $(\mathbb{R}, G_s)$ is a G-metric space.

Example 2.3 (see[1]). *Let $X = \{a, b\}$. Define G on $X \times X \times X$ by*

$$G(a, a, a) = G(b, b, b) = 0,$$
$$G(a, a, b) = 1, G(a, b, b) = 2.$$

Then it is clear that (X, G) is a G-metric space.

Definition 2.4 (see[1]). *Let (X, G) be a G-metric space, and let $\{x_n\}$ be a sequence of points of X, a point x inX is said to be the limit of the sequence $\{x_n\}$, if $\lim_{n,m\to\infty} G(x, x_n, x_m) \to 0$, and we say that the sequence $\{x_n\}$ is G-convergent to x or $\{x_n\}$ G-converges to x.*

Thus, $x_n \to x$ in a G-metric space (X, G) if for any $\epsilon > 0$, there exists $k \in \mathbb{N}$ such that $G(x, x_n, x_m) < \epsilon$ for all $m, n \geq k$.

Proposition 2.5 (see[1]). *Let (X, G) be a G-metric space. Then the following are equivalent.*
(1) $\{x_n\}$ is G-convergent to x.
(2) $G(x_n, x_n, x) \to 0$ as $n \to +\infty$.
(3) $G(x_n, x, x) \to 0$ as $n \to +\infty$.
(4) $G(x_n, x_m, x) \to 0$ as $n, m \to \infty$.

Definition 2.6 (see[1]). *Let (X, G) be a G-metric space a sequence $\{x_n\}$ is called G-Cauchy if for every $\epsilon > 0$, there is $k \in \mathbb{N}$ such that $G(x_n, x_m, x_l) < \epsilon$, for all $n, m, l \geq k$ that is, $G(x_n, x_m, x_l) \to 0$ as $n, m, l \to +\infty$.*

Definition 2.7 (see[1]). *A G-metric space (X, G) is called G-complete if every G-Cauchy sequence in (X, G) is G-convergent in (X, G).*

Proposition 2.8 (see[1]). *Let (X, G) be a G-metric space. Then the following are equivalent.*

(i) *The sequence $\{x_n\}$ is $G - Cauchy$.*

(ii) *For every $\epsilon > 0$, there is $k \in \mathbb{N}$ such that $G(x_n, x_m, x_m) < \epsilon$, for all $n, m \geq k$.*

3 Open Ball in G-metric Space

Now we define the notion of open ball in G-metric space as follows.

Definition 3.1. *Let (X, G) be a G-metric space, $x_0 \in X$, $r > 0$, then $B(x_0, r)$ is called G-open or open ball centered at x_0 with radius r if*

$$B(x_0, r) = \{x_0\} \cup \left\{ x \in X : \sup_{y \in B^*(x_0,r)} G(x_0, x, y) < r \right\}$$

where $B^*(x_0, r) = \{x \in X : G(x_0, x, x) < r\}$

Definition 3.2. *A set U in a G-metric space is said to be open if it contains a open ball. i.e. given any point $x \in U$, there exists $r > 0$ and an open ball $B(x, r)$ such that $B(x, r) \subset U$.*

Remark 3.3. *If $0 < r_1 < r_2$ then*
(i) $B^*(x_0, r_1) \subset B^*(x_0, r_2)$
(ii) $B(x_0, r_1) \subset B(x_0, r_2)$

Lemma 3.4. *Every G-open ball $B(x_0, r), x \in X, r > 0$ is an open set in X.*
Proof: Consider the ball $B(x_0, r)$ in X and let $x \in B(x_0, r)$ then

$$\sup_{a \in B^*(x_0,r)} G(x_0, x, a) = r_1 < r \quad and \quad G(x_0, a, a) = r_2 < r.$$

Now we choose $0 < r_0 = \max\{r_1, r_2\} < r$ then

$$\sup_{a \in B^*(x_0,r)} G(x_0, a, a) \leq r_0 < r$$

This implies $x \in B(a, r_0) \subset B(x_0, r)$ This proves that $B(x_0, r)$ is an open set in X.

4 G-metric Topology

In this section we discuss the topology on G-metric space X. We show that the collection $\beta = \{B(x_0, r) : x \in X, r > 0\}$ of G-open balls induces a topology on X, called G-metric topology.

Theorem 4.1. *The collection β of all G-open balls forms a basis for a topology τ on X.*
Proof. Let τ be a topology on X. To show that the collection β is a basis for τ it is enough to show that the collection β satisfies the following conditions:
(i) Obviously, $X \subset (\cup_{x \in X} B(x, r))$ and
(ii) if $x \in B(x_1, r) \cap B(x_2, r), r > 0$ for some $x_1, x_2 \in X$, then

$$\sup_{a \in B^*(x_1,r)} G(x_1, x, a) = s_1 < r \text{ and } \sup_{b \in B^*(x_2,r)} G(x_1, x, b) = s_2 < r$$

We choose $0 < s = \max\{s_1, s_2\} < r$ then from remark 3.3 we have

$$B(x, s) \in B(x_1, r) \cap B(x_1, r)$$

The collection β is a basis for τ .

Thus the G-metric space X together with a topology τ generated by G-metric is called a G-metric topological space and τ is called G-metric topology on X.

A topological space X is called G-metrizable if there exists a G-metric G on X that induces a topology on X. A G-metric space X is G-metrizable space together with the specific G-metric that induces the topology of X.

A set U is G-open in X in the G-metric topology τ induced by G-metric G if and only if for each $x \in U$, there is a $r > 0$ such that $B(x,r) \subset U$. Similarly, a set V is called τ-closed if its compliment $X \setminus V$ is τ-open.

Lemma 4.2. *A subset U of (X,τ) is G-open if and only if for any $x \in U$ there are finite real numbers $r_1, r_2, ..., r_n > 0$ such that*

$$x \in B(x,r_1) \cap B(x,r_2) \cap ... \cap B(x,r_n) \subset U$$

Proof. Since finite intersection of open ball is open, i.e.

$$B(x,r_1) \cap B(x,r_2) \cap ... \cap B(x,r_n)$$

is open and hence U is G-open.

Conversely, let U be G-open and $x \in U$. Consider a finite number of G- balls $B(x_i,r_i)$ for $i = 1,2,3,...,m$ such that

$$x \in B(x_i,r_i) \subset U$$

Since $x \in B(x_i,r_i)$ so $G(x,x_i,y) = s_i < r_i$, where $y \in B^*(x_i,r_i)$ for $i = 1,2,3,...,m$. Choose $t_i < \frac{r_i - s_i}{2}$. Then $B(x,t_i) \cap B(x_i,t_i) \in B(x_i,r_i)$ holds for all $i = 1,2,3,...,m$. So

$$\begin{aligned} & x \in B(x,t_1) \cap B(x_1,t_1) \cap B(x,t_2) \cap B(x_2,t_2) \cap B(x,t_m) \cap B(x_m,t_m) \\ & \subset \bigcap_{i=1}^{m} B(x_i,r_i) \subset U. \end{aligned}$$

This proves the lemma.

Theorem 4.3. *Arbitrary union and finite intersection of open balls $B(x,r)$, $x \in X$ are open.*

Proof. Let $x \in \cup B(x_i,r_i) \in U$ then for some i, $x \in B(x_i,r_i) \in U$ hence U is open.

Definition 4.4. *A set U in a G-metric space X, is said to be closed if its complement $X - U$ is τ-open.*

Theorem 4.5. *Finite union and arbitrary intersection of closed balls in a G-metric space are closed.*

Proof. Suppose $B = \cup_{i=1}^{n} B[x_i,r_i]$, then it is sufficient to show that complement of $B = C(B)$ is open. $C(B) = C(\cup_{i=1}^{n} B[x_i,r_i]) = \cap_{i=1}^{n} C(B[x_i,r_i])$ is open. Hence B is closed.

Now let $B[x_i,r_i]$ be the collection of closed bolls and $B = \cap B[x_i,r_i]$.
Consider $C(B) = C(\cap B[x_i,r_i]) = \cup C(\cap B[x_i,r_i])$ is open. Hence B is closed.

Definition 4.6. *$\overline{A}$ is called the G-closure of A if it is the intersection of all G-closed sets containing A*

$$\overline{B^*(x_0,r)} = \left\{x \in X : G(x_0,x,x) \leq r\right\} \text{ is the closure of } B^*(x_0,r) \text{ and}$$
$$\overline{B(x_0,r)} = \left\{x \in X : \sup_{y \in B^*(x_0,r)} G(x_0,x,y) \leq r\right\} \text{ is the closure of } B(x_0,r).$$

Remark 4.7. *It is clear that $B^*(x_0,r) \subset \overline{B^*(x_0,r)}$ and $B(x_0,r) \subset \overline{B(x_0,r)}$.*

Lemma 4.8. *If there exist a point $x \in B(x_0, r)$ with $G(x_0, x, x) = r_1 < r$, then $\overline{B(x_0, r_1)} \subset B(x_0, r)$.*

Proof: For $x \in B(x_0, r)$ with $G(x_0, x, x) = r_1 < r$. Let

$$\begin{aligned} x \in \overline{B(x_0, r_1)} &= \{z \in X : \sup_{x \in B*(x_0, r_1)} G(x_0, x, x) \leq r_1\} \\ &\subseteq \{z \in X : \sup_{x \in B*(x_0, r)} G(x_0, x, x) < r\} \\ &= B(x_0, r). \end{aligned}$$

Definition 4.9. *If A nonenpty subset of X, and $x \in (X, G)$ is said to be G-limit point of A, for any G-open set U containing x, there exist*

$$A \cap U - \{x\} = \phi.$$

Definition 4.10. *A sequence $\{x_n\}$ in a G-metric space (X, G) is said to be convergent (or G-convergent) if there exists an element x in X, for given $\epsilon > 0$ there exists $n_0 \in N$ such that $G(x_m, x_n, x) < \epsilon$ for all $m, n \geq n_0$.*
In such a case, it is said that $\{x_n\}$ converges to x and x is a limit point of $\{x_n\}$ and we write $x_n \to x$.

Lemma 4.11. *A sequence $\{x_n\}$ is convergent to x in (X, G) if and only if for any G-open set U containing x there exists a positive integer m such that $x_n \in U$ for all $n \geq m$.*

Proof. Suppose $x \in X$ and $\epsilon > 0$, since $x \in B(x, \epsilon)$ is G-open there exist $m \in \mathbb{N}$ such that $x_n \in B(x, \epsilon)$, for all $n \geq m$ gives $G(x_n, x_n, x) < \epsilon$ for $n \geq m$ i.e. $G(x_n, x_n, x) \to 0$ as $n \to \infty$. Thus $\{x_n\}$ converges to x in (X, G).

Conversely, suppose $\{x_n\}$ converges to x in (X, G). Let U be a G- open set with $x \in U$. From Lemma 4.2, we have

$$x \in B(x, r_1) \cap B(x_1, r_1) \cap B(x, r_1) \cap B(x_1, r_1) \cap \cdots \cap B(x, r_k) \cap B(x_k, r_k) \subset U.$$

For some $x_1, x_2, \cdots, x_k \in X$ and $r_1, r_2, \cdots, r_k > 0$. Since $G(x_n, x_k, x) \to 0$ as $n \to \infty$, there exists $m_k \in N$ such that $G(x_n, x, x_k) < r_k$ for all $n \geq m_k$ i.e. $x_n \in B(x, r_k)$ for all $n \geq m_k$ and this is true for each $i = 1, 2, \cdots k$. Taking $m = \max\{r_1, r_2 \cdots, r_k\}$ we obtain

$$\begin{aligned} x_n \in & B(x, r_1) \cap B(x_1, r_1) \cap B(x, r_1) \cap B(x_1, r_1) \cap \cdots \\ & \cap B(x, r_k) \cap B(x_k, r_k) \subset U, \quad \forall n \geq m. \end{aligned}$$

Thus the lemma is proved.

Definition 4.12. *$A \subset X$ is said to be dense in X if $\bar{A} = X$.*

Definition 4.13. *$A \subset X$ is said to be no-where dense if int$(\bar{A}) = \phi$ where interior of a set B is defined to be the union of all G-open sets contained in B.*

5 Cantor's and Baire's Theorem in G-metric Spaces

We define for $A \subset X$, and

$$\delta_a(A) = \sup_{a \in X}\{G(x, y, a) : x, y \in A\}$$

The term $\delta_a(A)$ need not be considered as the diameter of A.

Lemma 5.1. *For any $A \subset X$ and $a \in X$, then $\delta_a(A) = \delta_a(\bar{A})$*

Proof: Since $A \subset \bar{A}$ it follows that $\delta_a(A) \leq \delta_a(\bar{A})$. Now let $x, y \in \bar{A}$. If both $x, y \in A$, then clearly $G(x, y, a) \leq \delta_a(A)$. If suppose first that one of them, say, $x \notin A$ but $y \in A$. Let $0 < \epsilon_1 < \epsilon$ and since $x \in \bar{A}$ there exists z such that $z \in A \cap B(x, \epsilon_1)$ and this also implies $z \in A \cap B(x, \epsilon)$ as $z \in A \cap B(x, \epsilon_1) \subset A \cap B(x, \epsilon)$. Therefore

$$\begin{aligned} G(x, y, a) &\leq G(x, z, z) + G(z, y, a) \\ &\leq G(y, z, a) + \epsilon \end{aligned}$$

Since this is true for every $\epsilon > 0$, we conclude that

$$G(x, y, a) \leq \delta_a(A) \quad for \quad y \in A \quad and \quad x \in \bar{A}.$$

Finally, if $x, y \in \bar{A} - A$ then repeating the same argument we can show that in this case also $G(x, y, a) \leq \delta_a(A)$. Hence

$$\delta_a(\bar{A}) = \sup\{G(x, y, a) : x, y \in \bar{A}\} \leq \delta_a(A)$$

so $\delta_a(A) = \delta_a(\bar{A})$.

Theorem 5.2. ***Cantor's Intersection Throrem*** *Suppose that (X, G) is a complete G-metric space. If $\{F_n\}$ is any decreasing sequence of G-closed sets with $\delta_a(F_n) \to 0$ as $n \to \infty$, $\forall a \in X$ then $\bigcap_n F_n$ is non-empty and contains at most one point.*

Proof: Let x_n be a point of F_n, for each positive integer n. First we show $\{x_n\}$ is Cauchy in X. Since $\{F_n\}$ is decreasing, $x_m \in F_n$ for all $m \geq n$. Now for any arbitrary $a \in X$, $m \geq n$

$$G(x_m, x_n, a) \leq \delta_a(F_n) \to 0$$

as $n \to \infty$. This shows that $\{x_n\}$ is a Cauchy sequence in X. Since X is complete, there is $x \in X$ such that $x_n \to x$. Now our aim to show that $x \in \cap Fn$. If $x_k \neq x$ from some k onward, otherwise there is nothing to prove. Let $n \in N$ be fixed. Let U be any G-open set containing x. By Lemma 4.11, there is $n_1 \in N$ such that $x_k \in U$, for all $k \geq n_1$. Then $x_k \in [U - \{x\}] \cap F_n$, for all $k \geq \max\{n, n_1\}$. This shows that $x \in \bar{F_n} = F_n$, since F_n is G-closed. As this is true for all $n \in N$, $x \in \bigcap F_n$.

Now we have to show $\bigcap F_n$ contains exactly one point. suppose that it contains two distinct points x and y. Choose $z \in X, z \neq x \neq y$.

$$G(x, y, z) \leq \delta_z(F_n), \quad \forall n \in N.$$

Since $\delta_z(F_n) \to 0$ as $n \to \infty$ gives $G(x, y, z) = 0$ i.e. $x = y = z$ which is a contradiction. Hence This proves $\cap F_n$ contains exactly one point.

Theorem 5.3. *If in a G-metric space (X, G), for any decreasing sequence of G- closed sets $\{F_n\}$ with $\delta_a(Fn) \to 0$ as $n \to \infty$ for all $a \in X$, $\bigcap_n F_n$ consists of a single point, then (X, G) is complete.*

Proof: Let $\{x_n\}$ be a Cauchy sequence in X. Let $F_n = \{x_n, x_{n+1}, \cdots\}$ for any $n \in N$. Then $F_n \supset F_{n+1}$ gives $\bar{F_n} \supset \bar{F}_{n+1}$, for all $n \in N$. So $\{F_n\}$ is a decreasing sequence of G-closed sets. For $a \in X$ and $\epsilon > 0$ be a arbitrary, there is $n_1 \in N$ such that

$$G(x_m, x_n, a) \leq \epsilon, \quad \forall m, n \geq n_1.$$

This gives $\delta_a(F_{n_1}) \leq \epsilon$ and so by Lemma 5.1 we can write $\delta_a(\bar{F}_{n_1}) \leq \epsilon$. Since $\{\bar{F_n}\}$ is decreasing, for $n \geq n_1$, $\delta_a(\bar{F_n}) \leq \delta_a(\bar{F_{n_1}}) \leq \epsilon$. Therefore $\delta_a(\bar{F_n}) \to 0$ as $n \to \infty$. Hence by the given condition, $\bigcap \bar{F_n} = \{x_0\}$, say. This gives that for any $a \in X$, $G(x_n, x_0, a) \leq \delta_a(\bar{F_n}) \to 0$ as $n \to \infty$ which implies $x_n \to x_0$ in X and hence (X, G) is complete.

Lemma 5.4. *For any $x_0 \in X$ and $r > 0$,*

$$C(x_0, r) = \{x_0\} \cup \{x \in X : \sup_{y \in C^\star(x_0,r)} G(x_0, x, y)\}$$

where $C^\star(x_0, r) = \{x \in X : G(x_0, x, x) \leq r\}$ then $C(x_0, r)$ is G-closed set.

Proof: We will show that no point outside $C(x_0, r)$ is a G-limit point of $C(x_0, r)$. Let $d \notin C(x_0, r)$. Then $G(x_0, y, d) > r$. If possible, let d be a G-limit point of $C(x_0, r)$. Let $\epsilon > 0$ be given. Since $B(x_0, \epsilon) \cap B(d, \epsilon)$ is a G-open set containing d, there exists $e \in C(x_0, r) \cap \Big(B(x_0, \epsilon) \cap B(d, \epsilon) - \{d\}\Big)$. Then

$$\begin{aligned} G(x_0, y, d) &\leq G(x_0, e, e) + G(e, y, d) \\ &\leq G(x_0, y, e) + G(e, y, d) \\ &< r + \epsilon \end{aligned}$$

Since $\epsilon > 0$, we have $G(x_0, y, d) \leq r$ which is a contradiction. Thus d cannot be a G-limit point of $C(x_0, r)$. Hence $C(x_0, r)$ contains all its G-limit points and so $C(x_0, r)$ is G-closed.

Theorem 5.5. ***Baire's Category Theorem***
A complete G-metric space (X, G) satisfying the condition $X \subset (\cup_{x \in X} B(x, r))$ for every pair of points $x, y \in X$, there exists a sequence of G-closed balls $\{B_n\}$ with center at x and with $\delta_a(B_n) \to 0$ as $n \to \infty$, for all $a \in X$.

Proof: Suppose if possible,

$$X = \bigcup_{n \in N} X_n = \bigcup_{n \in N} \bar{X_n}$$

where each X_n is no-where dense i.e. X_n does not contain any non-empty G-open set. Let U be any G-open set. Since X_1 is no-where dense, X_1 cannot contain U. So there exists $x_1 \in U$ such that $x_1 \notin X_1$. Since $U - \bar{X_1}$ is G-open and $x_1 \in U - \bar{X_1}$, by Lemma 4.2 there exists some real $r_1, r_2, \cdots, r_n$ all positive such that

$$x_1 \in V_1 \subset U - \bar{X_1}$$

where $V_1 = B(x_1, r_1) \cap B(x_1, r_2) \cap \cdots \cap B(x_1, r_n)$
Without any loss of generality, because of the condition $X \subset (\cup_{x \in X} B(x, r))$, we can choose $B(x_1, r_1)$ such that $\delta_a(B(x_1, r_1)) < 1$ for all $a \in X$. Then $\delta_a(V_1) < 1$ for all $a \in X$. Choose

$$U_1 = B(x_1, r_1/2) \cap \cdots \cap B(x_1, r_n/2)$$

Then by Lemma 5.4

$$U_1 \subset C(x_1, r_1/2) \cap \cdots \cap C(x_1, r_n/2) \subset V_1 \subset U - \bar{X_1}$$

and $\delta_a(\bar{U_1}) \leq \delta_a(V_1) < 1$, for all $a \in X$. Again since U_1 is G-open and X_2 is no-where dense, $U_1 - \bar{X_2} \neq \phi$. So there exists $x_2 \in U_1 - \bar{X_2}$. Continuing as above we can find a G-open set U_2 such that

$$x_2 \in U_2 \subset \bar{U_2} \subset U_1 - \bar{X_2}$$

and $\delta_a(\bar{U_2}) < 1/2$ for all $a \in X$.

Continuing in this way we obtain a sequence of G-closed sets $\{\bar{U_n}\}$ such that $\bar{U}_{n+1} \subset \bar{U_n}$ for all $n \in N$, $\delta_a(\bar{U_n}) < 1/n$ for all $a \in X$ i.e. $\delta_a(\bar{U_n}) \to 0$ as $n \to \infty$ for all $a \in X$. By Cantors Intersection Theorem 5.2, $\bigcap \bar{U_n}$ is non-empty and contains at most one point. Let $\bigcap \bar{U_n} = \{x_0\}$. Since $\bar{U_n} \cap \bar{X_n} = \phi$ for all $n \in N$, $x_0 \notin \bigcap \bar{X_n}$ which is a contradiction. This proves the theorem.

6 Conclusion

In this work I have proved results on open and closed ball in G-metrics space and using using theory of G-metric space proved Canter's Intersection and Baire's Category theorems.

Competing Interests

Author has declared that no competing interests exist.

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