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# Application of Quick Simplex Method (A New Approach) On Two Phase Method

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***Authors' contributions***

*This work was carried out in collaboration between both authors. Both authors read and approved the final manuscript.*

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## Abstract

In this article, we suggest a new approach while solving two phase simplex method. The method sometimes involves less iteration than in the Simplex Method or at the most an equal number because the method attempts to replace more than one basic variable simultaneously. While dealing with Two Phase Simplex Method a new method [1,2,3,4,5,6] (Quick Simplex Method) can be applied in Phase I and also in Phase II.

This has been illustrated by giving the solution of solving Two Phase Simplex Method problems. It is also shown that either the iterations required are the same or less but iterations required are never more than those of the Simplex Method.

*Keywords: Basic feasible solution; optimum solution; simplex method; key determinant; constraints; net evaluation.*

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# 1 Introduction

There are two methods to obtain the solution of the linear programming problem. These methods can be classified as:

(i) The Graphical method (ii) Simplex method.

The linear programming has its own importance in obtaining the solution of a problem where two or more activities complete for limited resources.

Mathematically we have to maximize the objective function cx
subject to A x = b $x \geq 0$ ,

where

| | |
|---|---|
| x = n x 1 | column vector |
| A = m x n | coefficient matrix |
| b = m x 1 | column vector |
| C = 1 x n | row vector |

and the columns of $A$ are denoted by $P_{1}, P_{2}, \ldots, P_{n}$.

There are various alternative approaches to solve Simplex methods and Two Phase Simplex Method [7], [8,9,10] .

In this article, a new approach Quick Simplex method [6] is suggested while solving Two Phase Simplex Method. New method can be apply in Phase I and in Phase II wherever applicable. The method sometimes involves less iteration than in the simplex method or at the most an equal number because the method attempts to replace more than one basic variable simultaneously.

It is known that the conventional simplex method is rather inconvenient in handling the degeneracy and cycling problems because here the choice of the vectors, entering and outgoing, plays an important role. The degeneracy occurs when there is a tie for outgoing vector. The possibility of cycling is crucial only if the current basic feasible solution has more than one variable zero. In the Simplex method when there arises a tie for entering the vector, the vector with the lowest index j, is selected.

In our method the problem of tie in most of the degeneracy problems is solved. The powerfulness of our method lies in getting rid of the tie in the degeneracy problems. In such situations our technique positively is more powerful and handy as well.

# 2 Quick Simplex Method

For the solution of any L.P.P. by simplex algorithm, the existence of an initial basic feasible solution is always assumed. The steps for the computation of an optimum solution are as follows:

**Step 1)** Check whether the objective functions of the given L.P.P. is maximized or minimized. If it is minimized, convert it into a problem of maximizing by using the result

Minimum z = - Maximum(-z)

**Step 2)** Check whether RHS of the constraints, $q_i$ ("i=1,2,...,m".) are non-negative. If any one $q_i$ is negative, then multiply the corresponding inequalities of the constraints by (-1), so as to get all $q_i$ (i=1,2,…,m.) non-negative.

**Step 3)** Convert all the inequalities of the constraints into equations by introducing slack and surplus variables in the constraints. Put the costs of these variables equal to zero. Artificial variables can be used if needed.

**Step 4)** Obtain an initial basic feasible solution to the problem in the form $X_B = B^{-1}Q$ and put it in the last column of the simplex table.

**Step 5)** Compute the net evaluations $z_j - c_j$ ( j=1,2,…, n.) by using the relation

$z_j - c_j = C_B P j - c_j$ where $P_j = B^{-1} a_j$

Here we assume that at least one $z_j - c_j$ is negative.

**Step 6)** Choose most negative net evaluation. Let it be $z_j - c_j$ .Calculate the ratios $\theta_{rj}$ for positive $a_{rj}$. If all the entries in jth column are negative then go for next most negative net evaluation. If only one net evaluation say $z_j - c_j$ is negative and all the entries in the jth column are negative , then the problem has unbounded solution and procedure ends there otherwise goto the next step.

**Step 7)** Choose minimum $\theta_{rj}$ say $\theta_{kj}$ . Break the tie arbitrarily if any.

**Step 8)** Element of simplex table at the chosen element i.e. $a_{kj}$ will be the pivotal element.

**Step 9)** If number of pivotal element is less than m, then check if any more negative net evaluation is left. If it is so go to step 7 ignoring row and column of pivotal element otherwise go to the next step.

**Step 10)** Variables corresponding to the columns of pivotal elements are the incoming variables in the basis and they replace the variables corresponding the row of the pivotal elements. Note the number of pivotal elements say r. Then define key determinant of order (r x r) by using sub matrix of matrix A whose diagonal elements are the pivotal elements.

**Step 11)** Calculate entries in $X_B$ column using new basis, formed by using vectors containing all the pivotal elements.

**Step 12)** If all the entries are non-negative then the above replacement of r variables is allowable and next step can be followed otherwise the variable corresponding to the negative value of the $X_B$ column should not be entered in the basis. In such case goto step number 15.

**Step 13)** Determine entries in the new simplex table by replacing the r variables simultaneously.

**Step 14)** Check the optimality of the solution using net evaluations.

a) If all net evaluations are non-negative then the solution is optimal and procedure ends there.
b) If at least one net evaluation is negative go to step 6.

**Step 15)** Check if basis changes in case there was a tie, by choosing alternative variable in the basis, otherwise instead of minimum $\theta_{rj}$ . Choose next minimum $\theta_{rj}$ for which $z_j - c_j$ is negative and $X_i$ was not in the basis. This should be done for all those variables for which value of $X_B$ column is obtained as negative. Now go to step 10 using this new basis.

# 3 Statement of the Problem-I

Use two phase simplex method to solve the following LPP

Maximize $Z = 5x_1 + 8x_2$

Subject to the constraints:

$3x_1 + 2x_2 \geq 3$
$x_1 + 4x_2 \geq 4$
$x_1 + x_2 \leq 5$
$x_1, x_2 \geq 0$

# 4 Solution of the Problem by Two Phase Simplex Method [11]

In this problem it is observed that an optimum basic feasible solution has been reached by conventional Simplex Method in three step in Phase I and again three steps in Phase II and the solution is $x_1=0$ and $x_2=5$, and Max Z = 40.

# 5 Here we Apply Quick Simplex Method for Phase-I

**Step (1)**: (Initial table)

| $C_B$ | **$X_B$** | **0** | **0** | **0** | **-1** | **0** | **-1** | **0** | | **R1** | **R2** |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | **$P_1$** | **$P_2$** | **$P_3$** | **$P_4$** | **$P_5$** | **$P_6$** | **$P_7$** | **$X_B$** | | |
| -1 | $x_5$ | $\mathbf{a_1=3}$ | $b_1=2$ | –1 | 1 | 0 | 0 | 0 | 3 | 1 | 3/2 |
| -1 | $x_6$ | $a_2=1$ | $\mathbf{b_2=4}$ | 0 | 0 | -1 | 1 | 0 | 4 | 4 | 1 |
| 0 | $x_7$ | $a_3=1$ | $b_3=1$ | 0 | 0 | 0 | 0 | 1 | **5** | **5** | **5** |
| | $z_j - c_j$ | –4 | -6 | 1 | 0 | 1 | 0 | 0 | | | |
| | | ↑ | ↑ | | ↓ | | ↓ | | | | |

Here we introduce $P_1$, $P_2$ Simultaneously and outgoing vectors are $P_4$,$P_6$

To find new values in $X_B$ column.

Here we can find $c_1^{**}$, $c_2^{**}$ and $c_3^{**}$ using following formula [6].

$$\text{Column } X_B = \begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix}$$

$$c_1^{**} = \frac{(-1)^1\begin{vmatrix} 2 & 3 \\ 4 & 4 \end{vmatrix}}{10} = 2/5, \qquad c_2^{**} = \frac{(-1)^2\begin{vmatrix} 3 & 3 \\ 1 & 4 \end{vmatrix}}{10} = \quad 9/10\ ,$$

$$c_3^{**} = \frac{\begin{vmatrix} 3 & 2 & 3 \\ 1 & 4 & 4 \\ 1 & 1 & 5 \end{vmatrix}}{10} = 37/10, \qquad R = \begin{vmatrix} 3 & 2 \\ 1 & 4 \end{vmatrix} = 10$$

$$\text{New } X_B = \begin{bmatrix} 2/5 \\ 9/10 \\ 37/10 \end{bmatrix}$$

Column $P_3 = \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix}$ :−

$$c_1^{**} = \frac{(-1)^1 \begin{vmatrix} 2 & -1 \\ 4 & 0 \end{vmatrix}}{10} = -\frac{2}{5}, \qquad c_2^{**} = \frac{(-1)^2 \begin{vmatrix} 3 & -1 \\ 1 & 0 \end{vmatrix}}{10} = 1/1,$$

$$c_3^{**} = \frac{\begin{vmatrix} 3 & 2 & -1 \\ 1 & 4 & 0 \\ 1 & 1 & 0 \end{vmatrix}}{10} = 3/10, \qquad R = \begin{vmatrix} 3 & 2 \\ 1 & 4 \end{vmatrix} = 10$$

$$\text{New} P_3 = \begin{bmatrix} -2/5 \\ 1/10 \\ 3/10 \end{bmatrix}$$

Column $P_4 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ :−

$$c_1^{**} = \frac{(-1)^1 \begin{vmatrix} 2 & 1 \\ 4 & 0 \end{vmatrix}}{10} = 2/5, \qquad c_2^{**} = \frac{(-1)^2 \begin{vmatrix} 3 & 1 \\ 1 & 0 \end{vmatrix}}{10} = -1/10$$

$$\text{New Column } P_4 = \begin{bmatrix} 2/5 \\ -1/10 \\ -3/10 \end{bmatrix}$$

Column $P_5 = \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix}$ :−

$$c_1^{**} = \frac{(-1)^1 \begin{vmatrix} 2 & 0 \\ 4 & -1 \end{vmatrix}}{10} = 1/5, \qquad c_2^{**} = \frac{(-1)^2 \begin{vmatrix} 3 & 0 \\ 1 & -1 \end{vmatrix}}{10} = -3/10,$$

$$c_3^{**} = \frac{\begin{vmatrix} 3 & 2 & 0 \\ 1 & 4 & -1 \\ 1 & 1 & 0 \end{vmatrix}}{10} = 1/10 \qquad R = \begin{vmatrix} 3 & 2 \\ 1 & 4 \end{vmatrix} = 10$$

$$\text{New Column } P_5 = \begin{bmatrix} 1/5 \\ -3/10 \\ 1/10 \end{bmatrix}$$

$$\text{Column } P_6 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} :-$$

$$c_1^{**} = \frac{(-1)^1 \begin{vmatrix} 2 & 0 \\ 4 & 1 \end{vmatrix}}{10} = -1/5, \qquad c_2^{**} = \frac{(-1)^2 \begin{vmatrix} 3 & 0 \\ 1 & 1 \end{vmatrix}}{10} = 3/10$$

$$c_3^{**} = \frac{\begin{vmatrix} 3 & 2 & 0 \\ 1 & 4 & 1 \\ 1 & 1 & 0 \end{vmatrix}}{10} = -1/10, \qquad R = \begin{vmatrix} 3 & 2 \\ 1 & 4 \end{vmatrix} = 10$$

$$\text{New Column } P_6 = \begin{bmatrix} -1/5 \\ 3/10 \\ -1/10 \end{bmatrix}$$

$$\text{Column } P_7 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} :-$$

$$c_1^{**} = \frac{(-1)^1 \begin{vmatrix} 2 & 0 \\ 4 & 0 \end{vmatrix}}{10} = 0, \qquad c_2^{**} = \frac{(-1)^2 \begin{vmatrix} 3 & 0 \\ 1 & 0 \end{vmatrix}}{10} = 0,$$

$$\mathbf{c_3^{**}} = \frac{\begin{vmatrix} \mathbf{3} & \mathbf{2} & \mathbf{0} \\ \mathbf{1} & \mathbf{4} & \mathbf{0} \\ \mathbf{1} & \mathbf{1} & \mathbf{1} \end{vmatrix}}{\mathbf{10}} = \mathbf{1} \qquad \mathbf{R} = \begin{vmatrix} \mathbf{3} & \mathbf{2} \\ \mathbf{1} & \mathbf{4} \end{vmatrix} = \mathbf{10}$$

$$\text{New Column } P_7 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

So we get direct third simplex table using above formulae.

**Step (3): Introduce $P_1$ and drop $P_4$**

| $C_B$ | $X_B$ | **0** | **0** | **0** | **-1** | **0** | **-1** | **0** | $X_B$ |
|---|---|---|---|---|---|---|---|---|---|
| | | $P_1$ | $P_2$ | $P_3$ | $P_4$ | $P_5$ | $P_6$ | $P_7$ | |
| 0 | $x_1$ | 1 | 0 | –2/5 | 2/5 | 1/5 | -1/5 | 0 | **2/5** |
| 0 | $x_2$ | 0 | 1 | 1/10 | -1/10 | -3/10 | 3/10 | 0 | 9/10 |
| 0 | $x_7$ | 0 | 0 | 3/10 | -3/10 | 1/10 | -1/10 | 1 | 37/10 |
| | $z_j - c_j$ | 0 | 0 | 0 | 1 | 0 | 1 | 0 | |

Since all $z_j - c_j \geq 0$, we go to **PHASE-II**

**Step (4):**

| $C_B$ | $X_B$ | 5 | 8 | 0 | | 0 | | 0 | $X_B$ | R1 | R2 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | $P_1$ | $P_2$ | $P_3$ | $P_4$ | $P_5$ | $P_6$ | $P_7$ | | | |
| 5 | $x_1$ | 1 | 0 | $b_1$=–2/5 | | **$a_1$=1/5** | | 0 | **2/5** | **-ve** | **2** |
| 8 | $x_2$ | 0 | 1 | **$b_2$=1/10** | | $a_2$-=3/10 | | 0 | 9/10 | 9 | -ve |
| 0 | $x_7$ | 0 | 0 | $b_3$=3/10 | | $a_3$=1/10 | | 1 | 37/10 | 37/3 | 37 |
| | $z_j - c_j$ | 0 | 0 | -6/5 | | -7/5 | | 0 | | | |
| | | ↓ | ↓ | ↑ | | ↑ | | | | | |

Here we introduce $P_3$ and $P_5$ simultaneously and outgoing vectors are $P_2$ and $P_1$ .

To find new values in $X_B$ column.

Here we can find $c_1^{**}$, $c_2^{**}$ and $c_3^{**}$ using following formula [12,6]

$$\text{Column } X_B = \begin{bmatrix} 2/5 \\ 9/10 \\ 37/10 \end{bmatrix}$$

$$c_1^{**} = \frac{(-1)^1 \begin{vmatrix} -2/5 & 2/5 \\ 1/10 & 9/10 \end{vmatrix}}{-1/10} = -4\,, \qquad c_2^{**} = \frac{(-1)^2 \begin{vmatrix} 1/5 & 2/5 \\ -3/10 & 9/10 \end{vmatrix}}{-1/10} = -3$$

$$c_3^{**} = \frac{\begin{vmatrix} 1/5 & -2/5 & 2/5 \\ -3/10 & 1/10 & 9/10 \\ 1/10 & 3/10 & 37/10 \end{vmatrix}}{-1/10} = 5\,, \qquad R = \begin{vmatrix} 1/5 & -2/5 \\ -3/10 & 1/10 \end{vmatrix} = -1/10$$

$$\text{New } X_B = \begin{bmatrix} -4 \\ -3 \\ 5 \end{bmatrix}$$

Here we found entries in $X_B$ Column are negative so here we have to apply conventional simplex method in step 4 only.

**Step (4): Introduce $P_5$ and drop $P_1$**

| $C_B$ | $X_B$ | 5 | 8 | 0 | | 0 | | 0 | | Ratio |
|---|---|---|---|---|---|---|---|---|---|---|
| | | $P_1$ | $P_2$ | $P_3$ | $P_4$ | $P_5$ | $P_6$ | $P_7$ | $X_B$ | |
| 0 | $x_5$ | 5 | 0 | –2 | | **1** | | 0 | 2 | **2** |
| 8 | $x_2$ | 3/2 | 1 | -1/2 | | 0 | | 0 | 3/2 | -ve |
| 0 | $x_7$ | -1/2 | 0 | 1/2 | | 0 | | 1 | 7/2 | 37 |
| | $z_j - c_j$ | 7 | 0 | -4 | | 0 | | 0 | | |
| | | | | ↑ | | | | ↓ | | |

**Step (5): Introduce $P_3$ and drop $P_7$**

| $C_B$ | $X_B$ | 5 | 8 | 0 | | 0 | | 0 | $X_B$ |
|---|---|---|---|---|---|---|---|---|---|
| | | $P_1$ | $P_2$ | $P_3$ | $P_4$ | $P_5$ | $P_6$ | $P_7$ | |
| 0 | $x_5$ | 3 | 0 | 0 | | **1** | | 4 | 16 |
| 8 | $x_2$ | 1 | 1 | 0 | | 0 | | 1 | 5 |
| 0 | $x_3$ | -1 | 0 | 1 | | 0 | | 2 | 7 |
| | $z_j - c_j$ | 3 | 0 | 0 | | 0 | | 8 | |

Since all $z_j - c_j \geq 0$, an optimum basic feasible solution has been reached.

$x_1$=0 and $x_2$ = 5,and Max Z = 40.

It is observed that we reached to solution in PHASE -1 in one step only by Quick Simplex Method while in conventional Simplex Method we reached to solution after 2 step. here we got advantage of Quick Simplex Method.

## 6 Statement of the Problem-II

Use two phase simplex method to solve the following LPP

$$\text{Maximize } Z = -2x_1 - x_2$$

Subject to the constraints:

$$3x_1 + x_2 \geq 3$$
$$4x_1 + 3x_2 \geq 6$$
$$x_1 + 2x_2 \geq 3, \quad x_j \geq 0$$

## 7 Problem Modified in the Standard Form

$$\text{Max } Z = 0x_1 + 0x_2 + 0x_3 - x_4 + 0x_5 - x_6 + 0x_7 - x_8$$

Subject to the constraints:

$$3x_1 + x_2 - x_3 + x_4 = 3,$$
$$4x_1 + 3x_2 - x_5 + x_6 = 6 \qquad ,$$
$$x_1 + 2x_2 - x_7 + x_8 = 3, \quad x_j \geq 0$$

## 8 Solution to the Problem Using Simplex Method

In this problem it is observed that an optimum basic feasible solution has been reached by conventional Simplex Method in four step in phase I and again in one step in phase II and solution is $x_2$= 3/5 and $x_2$ =6/5 and Max Z = -12/5.

# 9 Solution to the Problem Using Quick Simplex Method

**PHASE-I**

**Step (1): (Initial table)**

| $C_B$ | $X_B$ | 0 | 0 | 0 | -1 | 0 | -1 | 0 | -1 | | R1 | R 2 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | $P_1$ | $P_2$ | $P_3$ | $P_4$ | $P_5$ | $P_6$ | $P_7$ | $P_8$ | $X_B$ | | |
| -1 | $x_4$ | $\mathbf{a_1=3}$ | $b_1=1$ | –1 | **1** | 0 | 0 | 0 | 0 | 3 | **1** | 3 |
| -1 | $x_6$ | $a_3=4$ | $b_3=3$ | 0 | 0 | -1 | 1 | 0 | 0 | 6 | 3/2 | 2 |
| -1 | $x_8$ | $a_2=1$ | $\mathbf{b_2=2}$ | 0 | 0 | 0 | 0 | -1 | **1** | 3 | 3 | **3/2** |
| | $z_j - c_j$ | –8 | -6 | 1 | 0 | 1 | 0 | 1 | 0 | | | |
| | | ↑ | ↑ | | ↓ | | | | ↓ | | | |

Here we introduce $P_1$ and $P_2$ simultaneously and outgoing vectors are $P_4$ and $P_8$

To find new values in $X_B$ column. Using above formulae we find

$$\text{New } X_B = \begin{bmatrix} 3/5 \\ 0 \\ 6/5 \end{bmatrix}$$

Similarly We find other entries.

**Step (2):**

| $C_B$ | $X_B$ | 0 | 0 | 0 | -1 | 0 | -1 | 0 | -1 | $X_B$ |
|---|---|---|---|---|---|---|---|---|---|---|
| | | $P_1$ | $P_2$ | $P_3$ | $P_4$ | $P_5$ | $P_6$ | $P_7$ | $P_8$ | |
| 0 | $x_1$ | 1 | 0 | –2/5 | 2/5 | 0 | 0 | 1/5 | -1/5 | 3/5 |
| 0 | $x_6$ | 0 | 0 | 1 | -1 | -1 | 1 | 1 | -1 | 0 |
| 0 | $x_2$ | 0 | 1 | 1/5 | -1/5 | **0** | 0 | -3/5 | **3/5** | 6/5 |
| | $z_j - c_j$ | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 1 | |

Since all $z_j - c_j \geq 0$ we go to **PHASE-II**

**Step (2):**

| $C_B$ | $X_B$ | -2 | -1 | 0 | -1 | 0 | -1 | 0 | -1 | $X_B$ |
|---|---|---|---|---|---|---|---|---|---|---|
| | | $P_1$ | $P_2$ | $P_3$ | $P_4$ | $P_5$ | $P_6$ | $P_7$ | $P_8$ | |
| -2 | $x_1$ | 1 | 0 | –2/5 | 2/5 | 0 | 0 | 1/5 | -1/5 | 3/5 |
| -1 | $x_6$ | 0 | 0 | 1 | -1 | -1 | 1 | **1** | -1 | 0 |
| -1 | $x_2$ | 0 | 1 | 1/5 | -1/5 | 0 | 0 | -3/5 | 3/5 | 6/5 |
| | $z_j - c_j$ | 2 | 0 | -2/5 | 2/5 | 1 | 0 | -4/5 | 4/5 | |
| | | | | | | | ↓ | ↑ | | |

**Step (3): Introducing $P_7$ and dropping $P_6$**

| $C_B$ | $X_B$ | -2 | -1 | 0 | -1 | 0 | -1 | 0 | -1 | $X_B$ |
|---|---|---|---|---|---|---|---|---|---|---|
| | | $P_1$ | $P_2$ | $P_3$ | $P_4$ | $P_5$ | $P_6$ | $P_7$ | $P_8$ | |
| -2 | $x_1$ | 1 | 0 | –3/5 | 3/5 | 0 | 0 | 0 | 0 | 3/5 |
| 0 | $x_7$ | 0 | 0 | 1 | -1 | -1 | 1 | **1** | -1 | 0 |
| -1 | $x_2$ | 0 | 1 | 4/5 | -4/5 | -3/5 | 3/5 | 0 | 0 | 6/5 |
| | $z_j - c_j$ | 0 | 0 | 2/5 | 3/5 | 3/5 | 2/5 | 0 | 1 | |

Since all $z_j - c_j \geq 0$ , an optimum basic feasible solution has been reached.

$x_1 = 3/5$ and $x_2 = 6/5$ and Max Z = -12/5.

In this problem it is observed that we reached a solution in PHASE -I in one step only by Quick Simplex Method, to which we reached after 3 steps when conventional Simplex Method is used. Hence the number of iterations required are reduced by Quick Simplex Algorithm Methodology. This method found to be more convenient when introducing more than one vector simultaneously.

## 10 Statement of the Problem-III

Use two phase simplex method to solve the following LPP

Minimize $z = 3x_1 - 2x_2 + 4x_3$

Subject to the constraints:

$3x_1 + 5x_2 + 4x_3 \geq 7$
$6x_1 + x_2 + 3x_3 \geq 4$
$7x_1 - 2x_2 - x_3 \leq 10$
$x_1 - x_2 + 5x_3 \geq 3$
$4x_1 + 7x_2 - 2x_3 \geq 2$
$x_j \geq 0$

Max $z = -3x_1 + 2x_2 - 4x_3$

Subject to the constraints:

$3x_1 + 5x_2 + 4x_3 \geq 7$
$6x_1 + x_2 + 3x_3 \geq 4$
$7x_1 - 2x_2 - x_3 \leq 10$
$x_1 - x_2 + 5x_3 \geq 3$
$4x_1 + 7x_2 - 2x_3 \geq 2$
$x_j \geq 0$

## 11 Solution of the Problem by Two Phase Simplex Method

**PHASE –I**

Maximize $z = 0x_1 + 0x_2 + 0x_3 + 0x_4 - x_5 + 0x_6 - x_7 + x_8 + x_9 - x_{10} + x_{11} - x_{12}$

Subject to the constraints:

$3x_1 + 5x_2 + 4x_3 - x_4 + x_5 = 7$
$6x_1 + x_2 + 3x_3 - x_6 + x_7 = 4$
$7x_1 - 2x_2 - x_3 + x_8 = 10$
$x_1 - x_2 + 5x_3 - x_9 + x_{10} = 3$
$4x_1 + 7x_2 - 2x_3 - x_{11} + x_{12} = 2$
$x_j \geq 0$

## 12 Solution to the Problem Using Simplex Method

In this problem it is observed that an optimum basic feasible solution has been reached by conventional Simplex Method in five step in phase I and again in three step in phase II.

and solution is $x_1$=0 and $x_2$=1 , $x_3$=1and Max Z= -2.

## 13 Solution to the Problem Using Quick Simplex Method

**Step (1): (Initial table)**

| | | 0 | 0 | 0 | 0 | -1 | 0 | -1 | 0 | 0 | -1 | 0 | -1 | | Ratio1 | Ratio2 | Ratio3 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $C_B$ | $X_B$ | $P_1$ | $P_2$ | $P_3$ | $P_4$ | $P_5$ | $P_6$ | $P_7$ | $P_8$ | $P_9$ | $P_{10}$ | $P_{11}$ | $P_{12}$ | $x_B$ | | | |
| -1 | $x_5$ | $a_4=3$ | $b_4=5$ | $c_4=4$ | -1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 7 | 7/3 | 7/5 | 1.75 |
| -1 | $x_7$ | $\boldsymbol{a_1=6}$ | $b_1=1$ | $c_1=3$ | 0 | 0 | -1 | 1 | 0 | 0 | 0 | 0 | 0 | 4 | **2/3** | 4 | 1.33 |
| 0 | $x_8$ | $a_5=7$ | $b_5=-2$ | $c_5=-1$ | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 10 | 10/7 | -5 | -10 |
| -1 | $x_{10}$ | $a_2=1$ | $b_2=-2$ | $\boldsymbol{c_2=5}$ | 0 | 0 | 0 | 0 | 0 | -1 | 1 | 0 | 0 | 3 | 3 | -1.5 | **0.6** |
| -1 | $x_{12}$ | $a_3=4$ | $\boldsymbol{b_3=7}$ | $c_3=-2$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 | 1 | 2 | 1/2 | **0.28** | -1 |
| | $z_j-c_j$ | -14 | -11 | -10 | 1 | 0 | 1 | 0 | 0 | 1 | 0 | 1 | 0 | | | | |
| | | ↑ | ↑ | ↑ | | | | ↓ | | | ↓ | | ↓ | | | | |

Here we introduce $P_1, P_2, P_3$ simultaneously and outgoing vectors are $P_7, P_{12}, P_{10}$ .

To find new values in $X_B$ column.

Here we can find $d_1^{***}, d_2^{***}, d_3^{***}$ and $d_4^{***}$ using following formula.

$$R = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix},\ d_1^{***} = \begin{vmatrix} b_1 & c_1 & d_1 \\ b_2 & c_2 & d_2 \\ b_3 & c_3 & d_3 \end{vmatrix} \Big/ R$$

$$d_2^{***} = \begin{vmatrix} a_1 & d_1 & c_1 \\ a_2 & d_2 & c_2 \\ a_3 & d_3 & c_3 \end{vmatrix} \Big/ R\ , d_3^{***} = \begin{vmatrix} a_1 & b_1 & d_1 \\ a_2 & b_2 & d_2 \\ a_3 & b_3 & d_3 \end{vmatrix} \Big/ R$$

$$d_4^{***} = \begin{vmatrix} a_1 & b_1 & c_1 & d_1 \\ a_2 & b_2 & c_2 & d_2 \\ a_3 & b_3 & c_3 & d_3 \\ a_4 & b_4 & c_4 & d_4 \end{vmatrix} \Big/ R$$

$$\text{Column } X_B = \begin{bmatrix} 7 \\ 4 \\ 10 \\ 3 \\ 2 \end{bmatrix},$$

$$R = \begin{vmatrix} 6 & 1 & 3 \\ 1 & -2 & 5 \\ 4 & 7 & -2 \end{vmatrix} = -119$$

$$d_1^{***} = \begin{vmatrix} 1 & 3 & 4 \\ -2 & 5 & 3 \\ 7 & -2 & 2 \end{vmatrix} \Big/ R = 33/119, \; d_2^{***} = \begin{vmatrix} 6 & 4 & 3 \\ 1 & 3 & 5 \\ 4 & 2 & -2 \end{vmatrix} \Big/ R = 38/119,$$

$$d_3^{***} = \begin{vmatrix} 6 & 1 & 4 \\ 1 & -2 & 3 \\ 4 & 7 & 2 \end{vmatrix} \Big/ R = 80/119,$$

$$d_4^{***} = \begin{vmatrix} 6 & 1 & 3 & 4 \\ 1 & -2 & 5 & 3 \\ 4 & 7 & -2 & 2 \\ 3 & 5 & 4 & 7 \end{vmatrix} \Big/ R = 32/17, \; d_5^{***} = \begin{vmatrix} 6 & 1 & 3 & 4 \\ 1 & -2 & 5 & 3 \\ 4 & 7 & -2 & 2 \\ 7 & -2 & -1 & 10 \end{vmatrix} \Big/ R = 1115/119$$

Here we fill the entries according to the above table.

$$\text{New } X_B = \begin{bmatrix} 32/17 \\ 33/119 \\ 1115/119 \\ 38/119 \\ 80/119 \end{bmatrix}$$

In this way we can find other entries of respective columns. Thus we reached directly to fourth table of conventional simplex method.

**Step (4)**:

| | | 0 | 0 | 0 | 0 | -1 | 0 | -1 | 0 | 0 | -1 | 0 | -1 | | Ratio |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $C_B$ | $X_B$ | $P_1$ | $P_2$ | $P_3$ | $P_4$ | $P_5$ | $P_6$ | $P_7$ | $P_8$ | $P_9$ | $P_{10}$ | $P_{11}$ | $P_{12}$ | $x_B$ | |
| -1 | $x_5$ | 0 | 0 | 0 | -1 | 1 | -11/17 | | 0 | **29/17** | | 22/17 | | 32/17 | **32/29** |
| 0 | $x_3$ | 0 | 0 | 1 | 0 | 0 | 15/119 | | 0 | -38/119 | | -13/119 | | 80/119 | -ve |
| 0 | $x_8$ | 0 | 0 | 0 | 0 | 0 | 276/119 | | 1 | -247/119 | | -144/119 | | 1115/119 | -ve |
| 0 | $x_2$ | 0 | 1 | 0 | 0 | 0 | 22/119 | | 0 | -24/119 | | -27/119 | | 38/119 | -ve |
| 0 | $x_1$ | 1 | 0 | 0 | 0 | 0 | -31/119 | | 0 | 23/119 | | 11/119 | | 33/119 | 33/23 |
| | $z_j - c_j$ | 0 | 0 | 0 | 1 | 0 | 11/17 | | 0 | -29/17 | | -22/17 | | | |
| | | | | | | ↓ | | | | ↑ | | | | | |

**Step (5)**: Introduce $P_9$ and drop $P_5$

| | | 0 | 0 | 0 | 0 | -1 | 0 | -1 | 0 | 0 | -1 | 0 | -1 | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $C_B$ | $X_B$ | $P_1$ | $P_2$ | $P_3$ | $P_4$ | $P_5$ | $P_6$ | $P_7$ | $P_8$ | $P_9$ | $P_{10}$ | $P_{11}$ | $P_{12}$ | $x_B$ |
| 0 | $x_9$ | 0 | 0 | 0 | -17/29 | | -11/29 | | 0 | 1 | | 22/29 | | 32/29 |
| 0 | $x_3$ | 0 | 0 | 1 | -38/203 | | 1/203 | | 0 | 0 | | 27/203 | | 208/203 |
| 0 | $x_8$ | 0 | 0 | 0 | -247/203 | | 311/203 | | 1 | 0 | | 74/203 | | 2367/203 |
| 0 | $x_2$ | 0 | 1 | 0 | -24/203 | | 22/203 | | 0 | 0 | | -15/203 | | 110/203 |
| 0 | $x_1$ | 1 | 0 | 0 | 23/203 | | -38/203 | | 0 | 0 | | -11/203 | | 13/203 |
| | $z_j - c_j$ | 0 | 0 | 0 | 0 | | 0 | | 0 | 0 | | 0 | | |
| | | | | | | | | | | | | | | |

Since all $z_j - c_j \geq 0$, we go to PHASE-II

**Step (6)**:

| | | -3 | 2 | -4 | 0 | -1 | 0 | -1 | 0 | 0 | -1 | 0 | -1 | | Ratio |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $C_B$ | $X_B$ | $P_1$ | $P_2$ | $P_3$ | $P_4$ | $P_5$ | $P_6$ | $P_7$ | $P_8$ | $P_9$ | $P_{10}$ | $P_{11}$ | $P_{12}$ | $x_B$ | |
| 0 | $x_9$ | 0 | 0 | 0 | -17/29 | | -11/29 | | 0 | 1 | | **22/29** | | 32/29 | **16/11** |
| -4 | $x_3$ | 0 | 0 | 1 | -38/203 | | 1/203 | | 0 | 0 | | 27/203 | | 208/203 | 208/27 |
| 0 | $x_8$ | 0 | 0 | 0 | -247/203 | | 311/203 | | 1 | 0 | | 74/203 | | 2367/203 | 2367/74 |
| 2 | $x_2$ | 0 | 1 | 0 | -24/203 | | 22/203 | | 0 | 0 | | -15/203 | | 110/203 | |
| -3 | $x_1$ | 1 | 0 | 0 | 23/203 | | -38/203 | | 0 | 0 | | -11/203 | | 13/203 | |
| | $z_j - c_j$ | 0 | 0 | 0 | 5/29 | | 22/29 | | 0 | 0 | | -15/29 | | | |
| | | | | | | | | | | ↓ | | ↑ | | | |

Here we proceed as conventional simplex method to reach the solution as there is only one vector whose $z_j - c_j$ is negative.

**Step (7)**: Introduce $P_{11}$ and drop $P_9$

| | | -3 | 2 | -4 | 0 | -1 | 0 | -1 | 0 | 0 | -1 | 0 | -1 | | Ratio |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $C_B$ | $X_B$ | $P_1$ | $P_2$ | $P_3$ | $P_4$ | $P_5$ | $P_6$ | $P_7$ | $P_8$ | $P_9$ | $P_{10}$ | $P_{11}$ | $P_{12}$ | $x_B$ | |
| 0 | $x_{11}$ | 0 | 0 | 0 | -17/22 | | -1/2 | | 0 | 29/22 | | 1 | | 16/11 | -ve |
| -4 | $x_3$ | 0 | 0 | 1 | -13/154 | | 1/14 | | 0 | -27/154 | | 0 | | 64/77 | -ve |
| 0 | $x_8$ | 0 | 0 | 0 | -72/77 | | 12/7 | | 1 | -37/77 | | 0 | | 857/77 | -ve |
| 2 | $x_2$ | 0 | 1 | 0 | -27/154 | | 1/14 | | 0 | 15/154 | | 0 | | 50/77 | -ve |
| -3 | $x_1$ | 1 | 0 | 0 | **1/14** | | -3/14 | | 0 | 1/14 | | 0 | | 1/7 | 2 |
| | $z_j - c_j$ | 0 | 0 | 0 | -5/22 | | 1/2 | | 0 | 15/22 | | 0 | | | |
| | | ↓ | | | ↑ | | | | | | | | | | |

**Step (8)**: Introduce $P_4$ and drop $P_1$

| | | -3 | 2 | -4 | 0 | -1 | 0 | -1 | 0 | 0 | -1 | 0 | -1 | | Ratio |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $C_B$ | $X_B$ | $P_1$ | $P_2$ | $P_3$ | $P_4$ | $P_5$ | $P_6$ | $P_7$ | $P_8$ | $P_9$ | $P_{10}$ | $P_{11}$ | $P_{12}$ | $x_B$ | |
| 0 | $x_{11}$ | 119/11 | 0 | 0 | 0 | | -31/11 | | 0 | 23/11 | | 1 | | 3 | |
| -4 | $x_3$ | 13/11 | 0 | 1 | 0 | | -2/11 | | 0 | -1/11 | | 0 | | 1 | |
| 0 | $x_8$ | 144/11 | 0 | 0 | 0 | | -12/11 | | 1 | 5/11 | | 0 | | 13 | |
| 2 | $x_2$ | 27/11 | 1 | 0 | 0 | | 5/11 | | 0 | 3/11 | | 0 | | 1 | |
| 0 | $x_4$ | 14 | 0 | 0 | **1** | | -3 | | 0 | 1 | | 0 | | 2 | |
| | $z_j - c_j$ | 35/11 | 0 | 0 | 0 | | 2/11 | | 0 | 10/11 | | 0 | | | |
| | | | | | | | | | | | | | | | |

Since all $z_j - c_j \geq 0$, an optimum basic feasible solution has been reached.

$x_1$=0 and $x_2$=1, $x_3$=1 and Max Z= -2.

## 14 Conclusion

In the above problems it is observed that we reached to solution in PHASE -1 in one step only by quick simplex method .while in conventional simplex we reached to solution after 3 or more steps. Such advantage occurs by following Quick Simplex Method [6] in solving many linear programming problems.

## Competing Interests

Authors have declared that no competing interests exist.

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